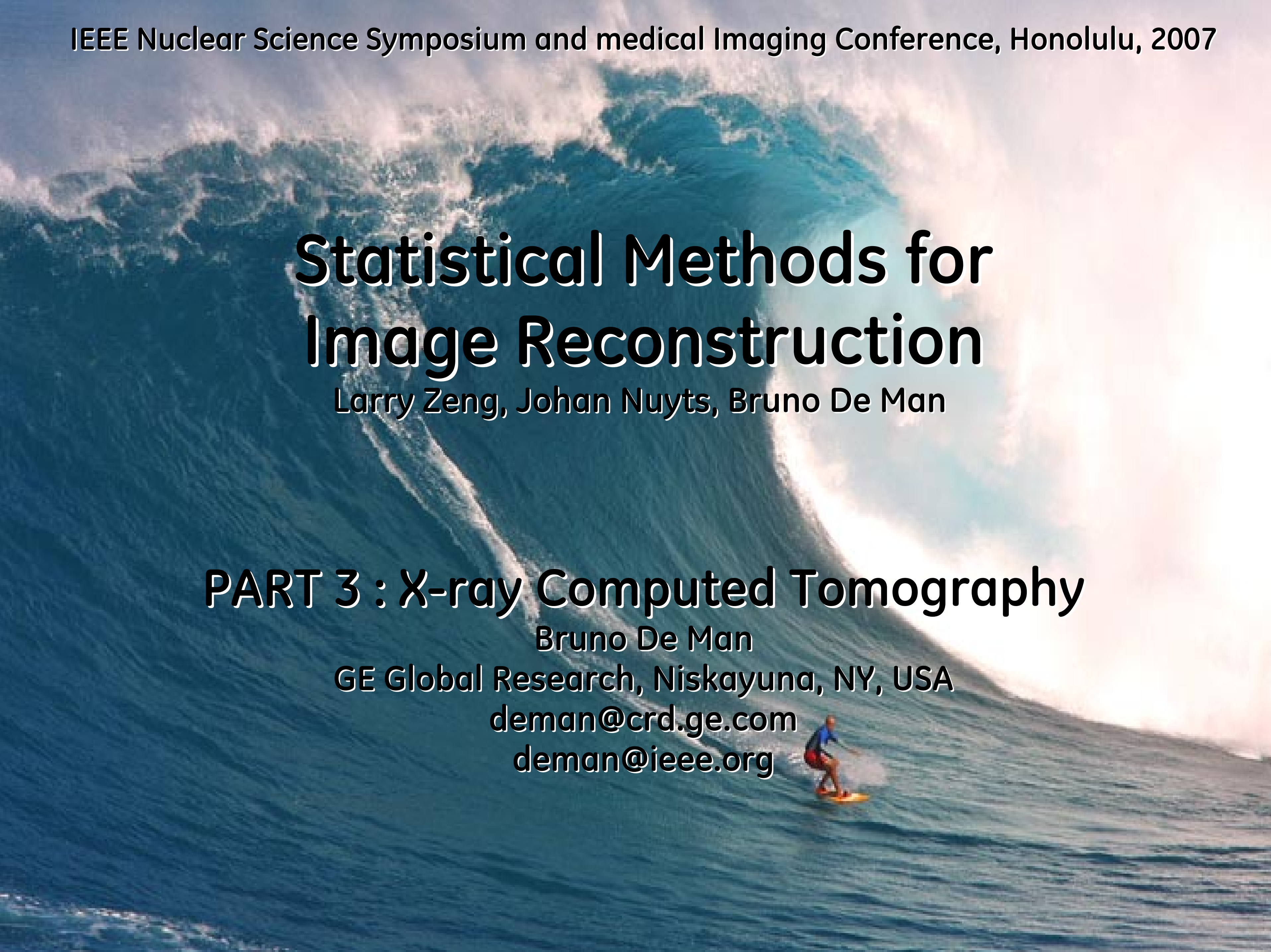

IEEE Nuclear Science Symposium and medical Imaging Conference, Honolulu, 2007

Statistical Methods for Image Reconstruction

Larry Zeng, Johan Nuyts, Bruno De Man

PART 3 : X-ray Computed Tomography

Bruno De Man
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Overview

- CT basics
- Noise models & cost functions
- Forward model – projector/backprojector
- Prior model
- Update step
- Image quality
- Advanced forward model - incorporate physics

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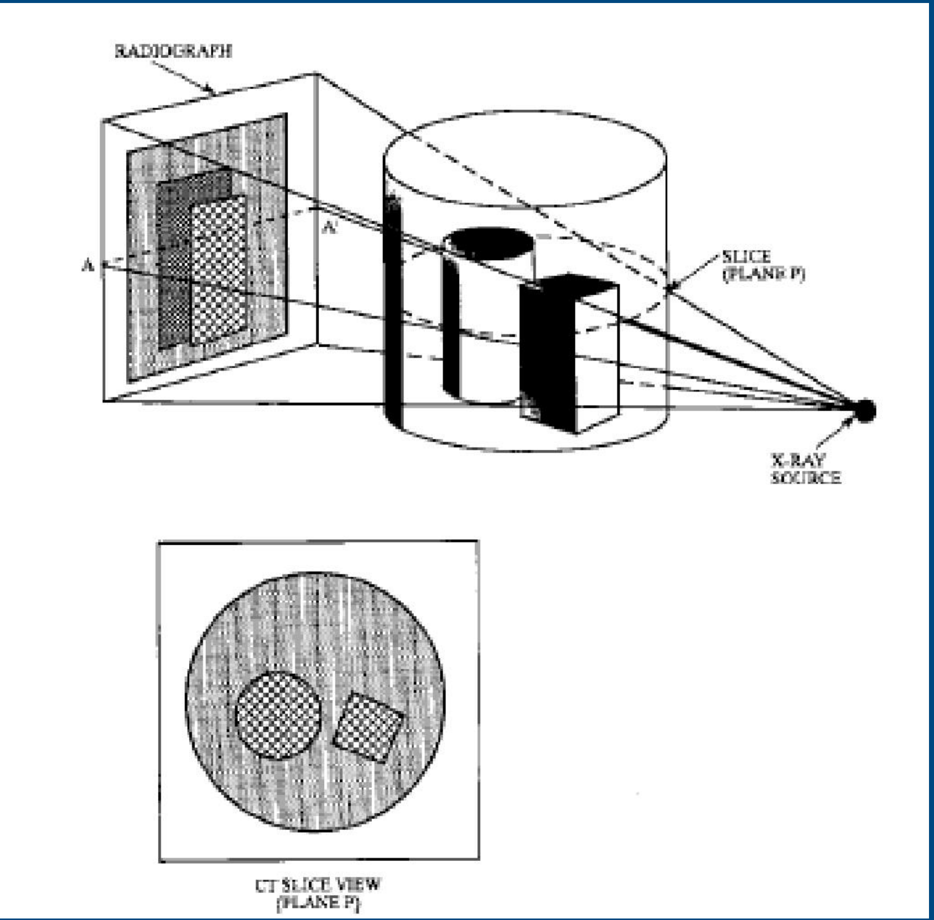
IEEE MIC, Honolulu, 2007

What is X-ray CT ?

1. Produce X-rays w/ X-ray tube
2. Pass x-rays through patient
3. Detect on the other side
4. Repeat from all angles surrounding patient
5. Reconstruct cross sectional images
6. Pixel values represent attenuation of tissue



http://www.slaney.org/pct/

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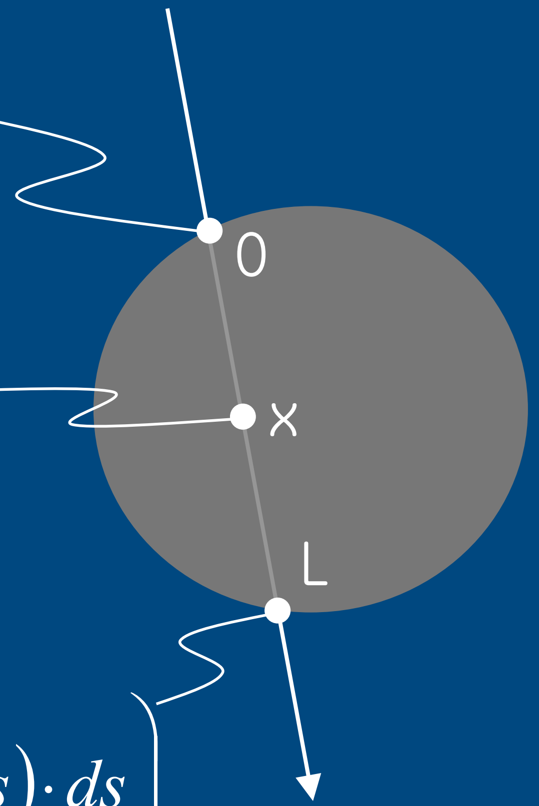
Beer's law

$$I(0) = I_0$$

$$I(x) = I_0 \cdot \exp\left(-\int_0^x \mu(s) \cdot ds \right)$$



$$I(L) = I_0 \cdot \exp\left(-\int_0^L \mu(s) \cdot ds \right)$$

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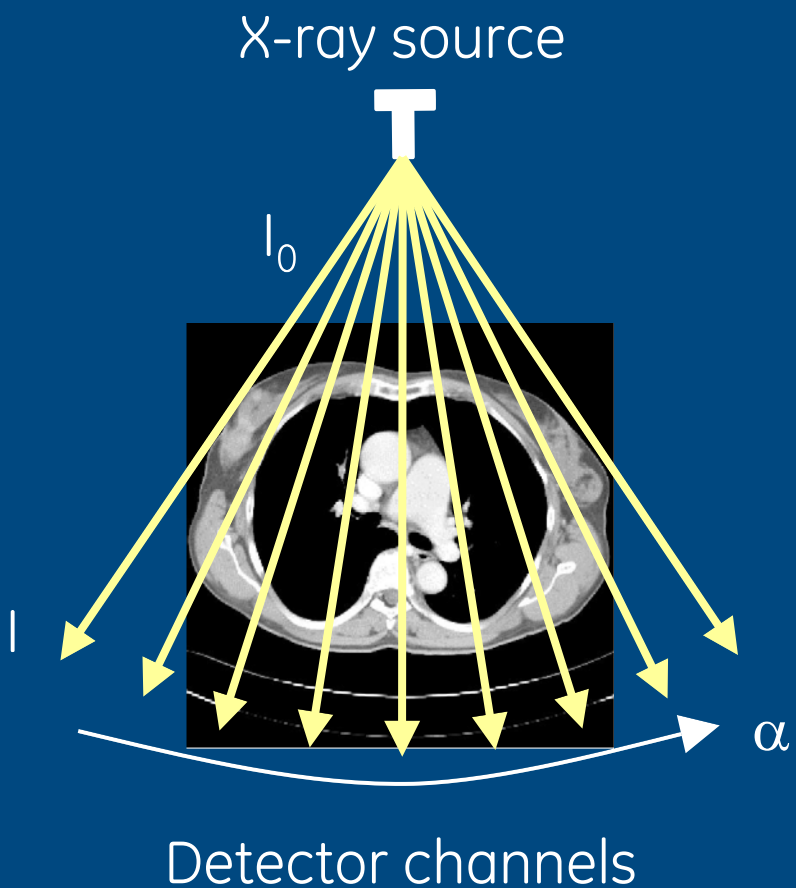
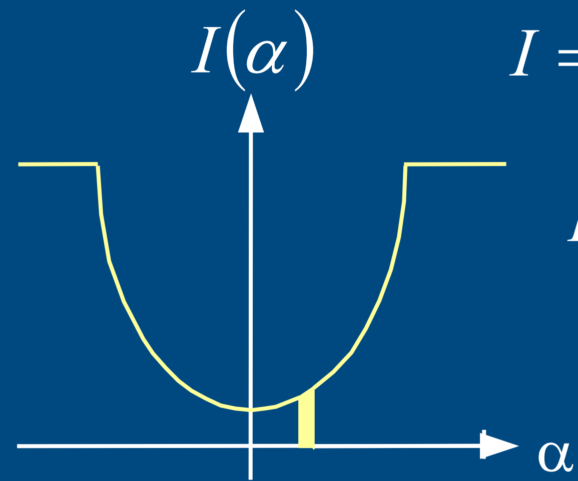
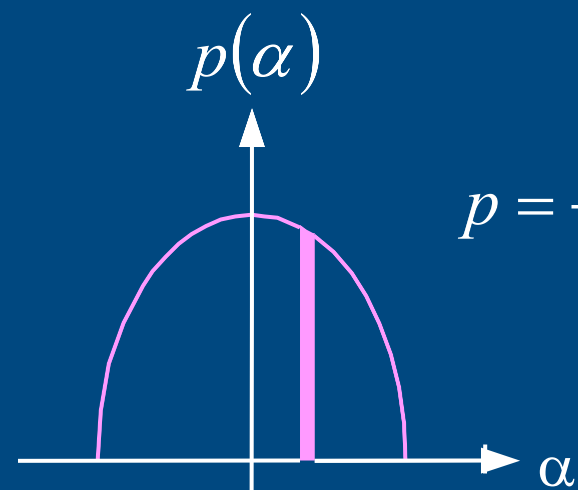
One “view” or “projection”





$$I = I_0 \cdot \exp(-p)$$

$$p = \int_0^L \mu(s) \cdot ds$$



$$p = -\ln\left(\frac{I}{I_0}\right)$$

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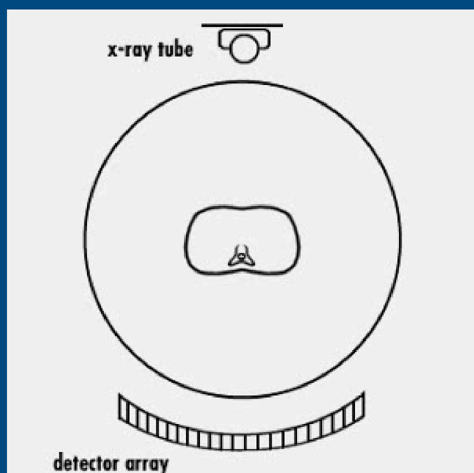
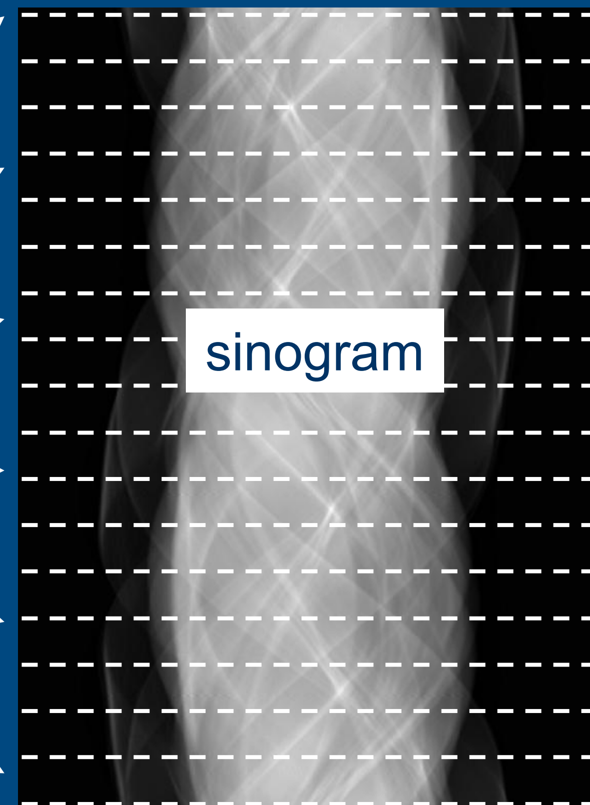
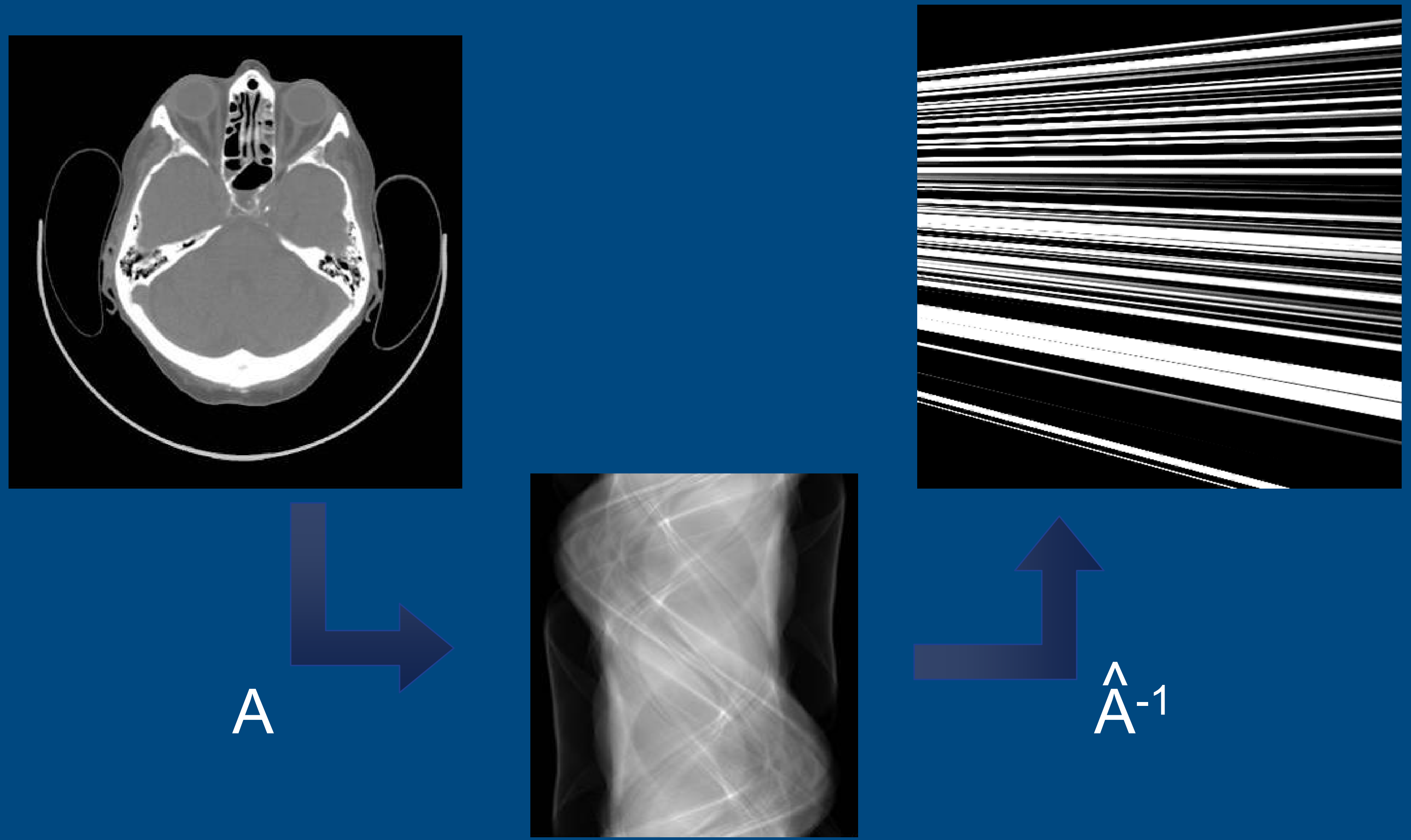
Sinogram

Stack all views or projections





Filtered backprojection



Shepp, IEEE Trans. Nucl. Sci., 1974

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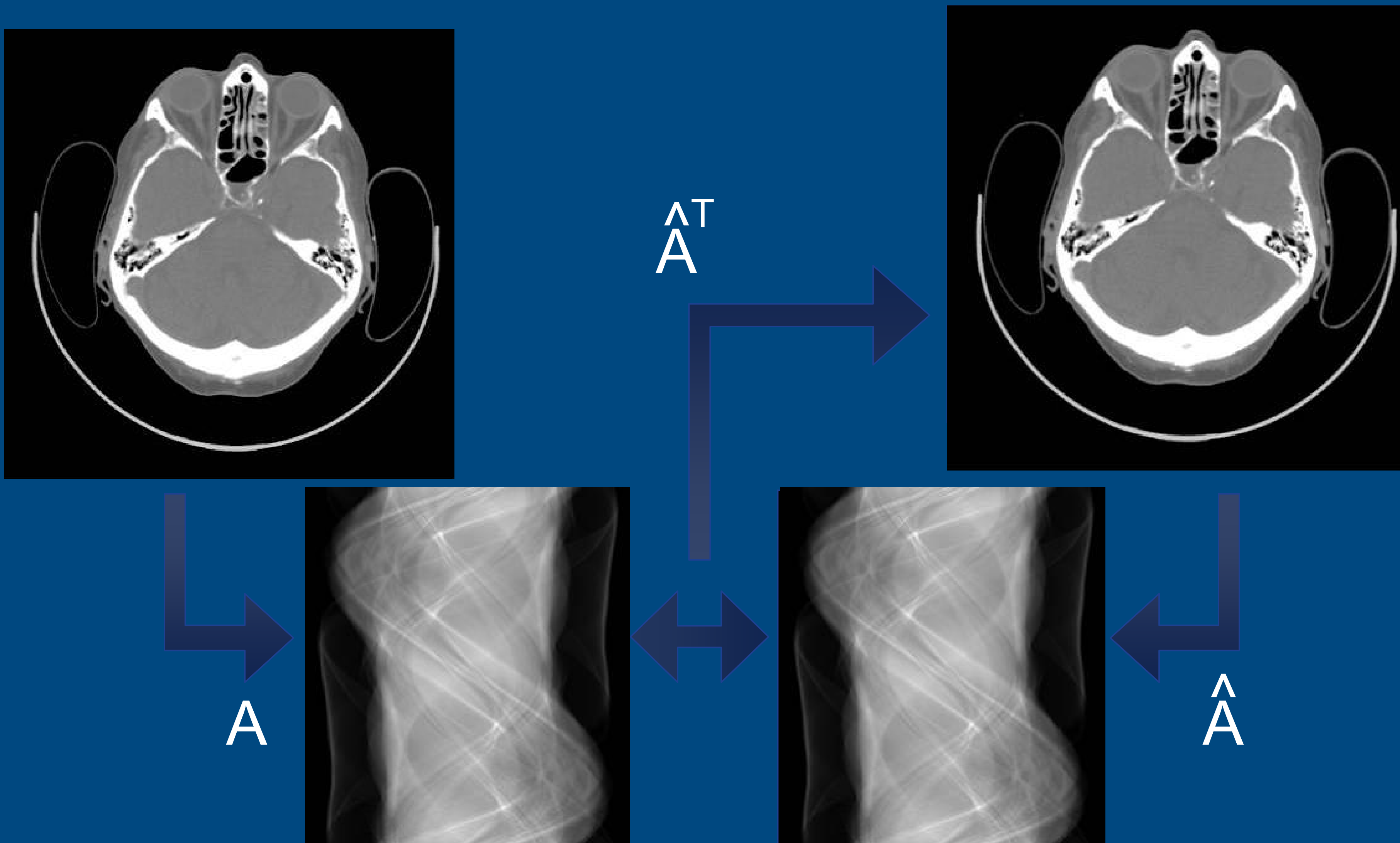
Iterative reconstruction



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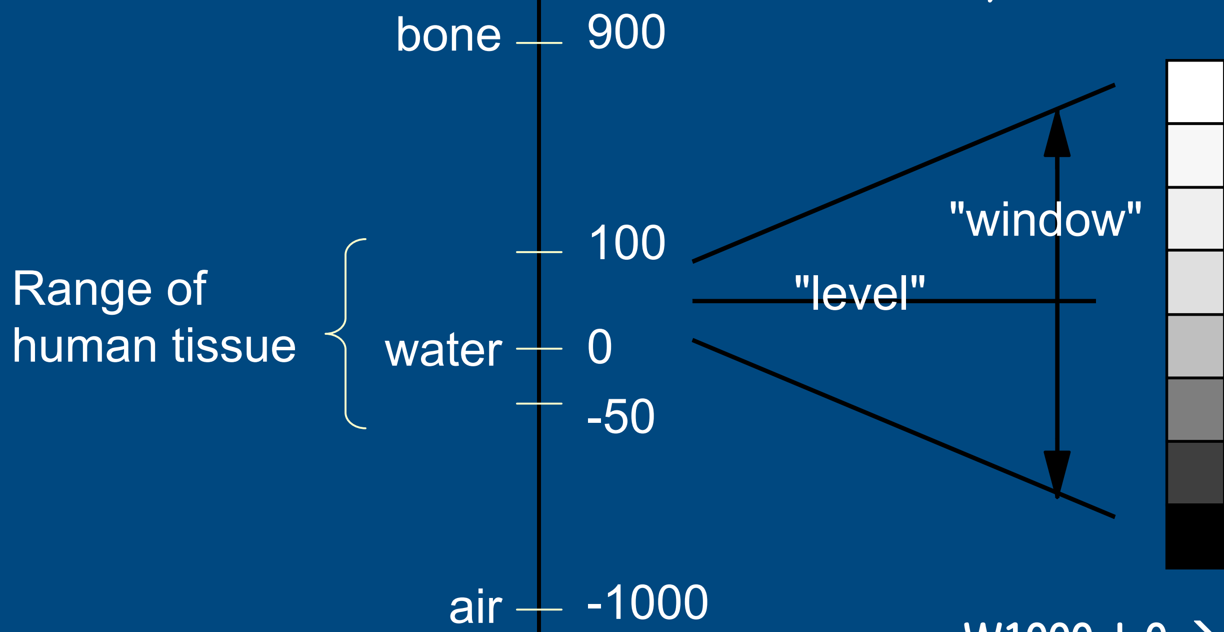
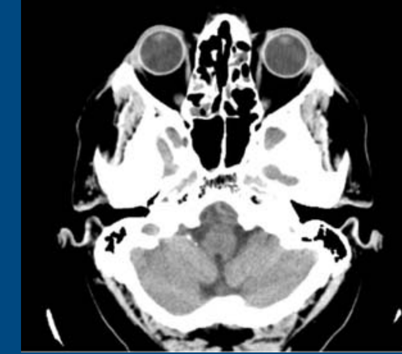
CT number - Hounsfield Units

CT number =
$(\mu - \mu_{H20}) / \mu_{H20} * 1000$

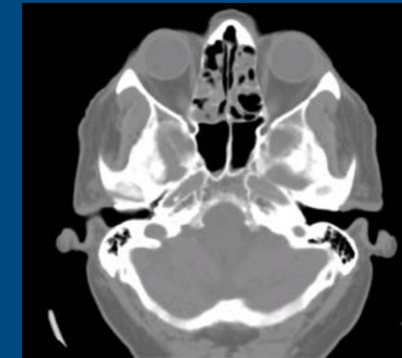
HU scale



W100, L 20 → Soft tissue contrast visible

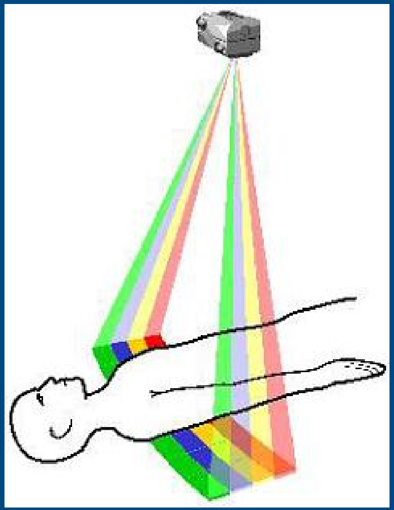


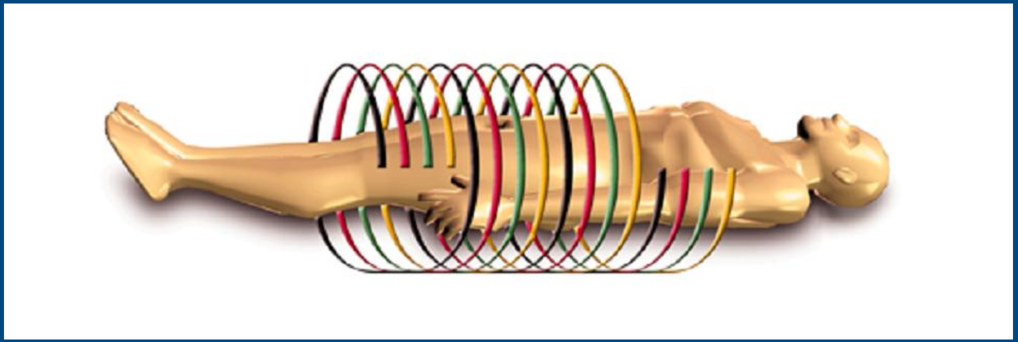
W1000, L 0 → Bone structure visible



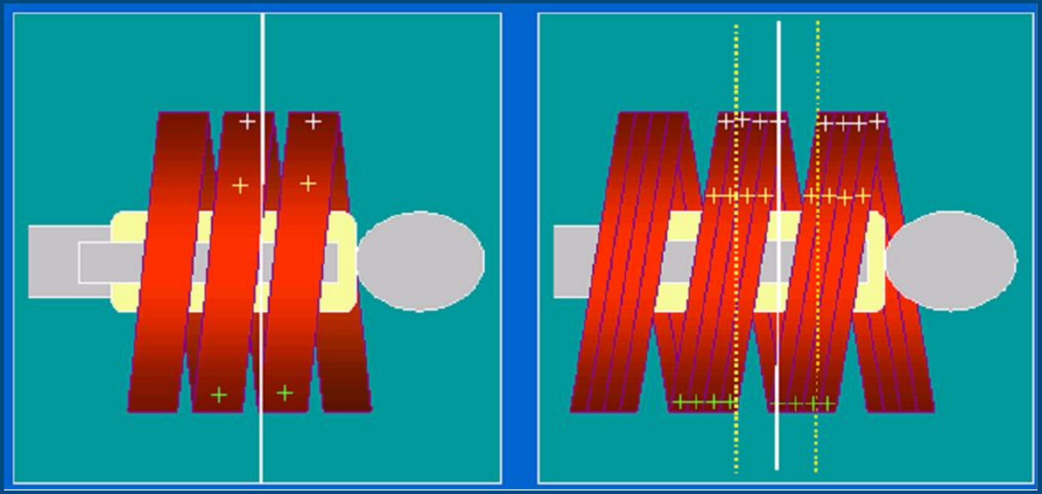
< Courtesy of Tom Toth (GE Healthcare) >

Multi-slice (Multi-detector-row) CT





Higher resolution – Larger coverage – Faster scanning



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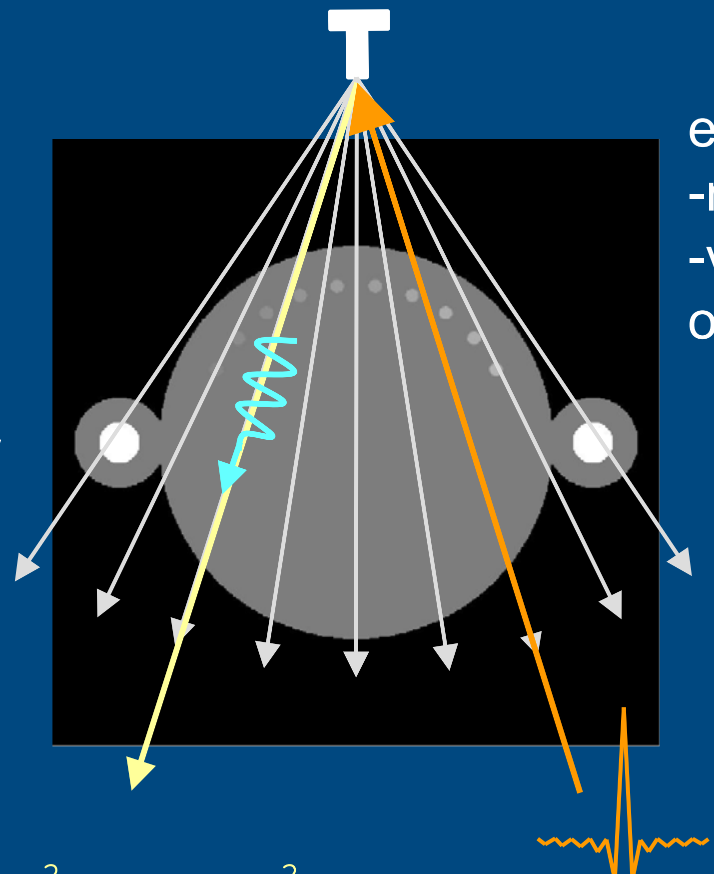
< Courtesy of ImPACT >

CT measurement noise

quantum noise :
-Poisson distribution for photon-counting
-variance equal to number of photons
-Compound-Poisson for energy-integrating



electronic noise :
-normal distribution
-variance is independent of signal strength

$$\sigma_I^2 = \sigma_{quantum}^2 + \sigma_{electronic}^2$$

$$\sigma_p$$

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Exercise 1

Assume :

- an x-ray flux of 1.e6 photons per channel per view (air scan)
- a 20cm water phantom with μ_{water}=0.2cm^{-1}
- a 1cm central low-contrast object with μ_{water}=0.21cm^{-1}
- a photon-counting detector with 100% detection efficiency

Question :

- what is the contrast-to-noise ratio in one single view ?
- in the intensity-domain ?
- and in the attenuation-domain ? (after log-conversion)

Tips

- Beer's law
- $\sigma_p = \sigma_I \cdot |\partial p/\partial I|$

Verify answers ? Please email.

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Bayesian framework

$$\arg\max_{\text{img}}\left[P(\text{img}\,|\,\text{meas})\right]$$

$$=\arg\max_{\text{img}}\left[\frac{P(\text{meas}\,|\,\text{img})\cdot P(\text{img})}{P(\text{meas})}\right]$$

$$=\arg\max_{\text{img}}\left[\log\frac{P(\text{meas}\,|\,\text{img})\cdot P(\text{img})}{P(\text{meas})}\right]$$

$$=\arg\max_{\text{img}}\left[\underbrace{\log P(\text{meas}\,|\,\text{img})}_{\textbf{LIKELIHOOD}}+\underbrace{\log P(\text{img})}_{\textbf{PRIOR}}\right]$$

ML

MAP

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Bayesian framework

$$\arg\max_{\text{img}} [\underbrace{\log P(\text{meas} \mid \text{img})}^{\text{LIKELIHOOD}} + \underbrace{\log P(\text{img})}^{\text{PRIOR}}]$$

$$\left.\begin{array}{lll} \text{Forward model} & \longrightarrow & \text{calc} = \text{f(img)} \\ \text{Noise model} & \longrightarrow & P(\text{meas} \mid \text{calc}) \end{array}\right\} P(\text{meas} \mid \text{img})$$

Optimization method $\longrightarrow$ Update step

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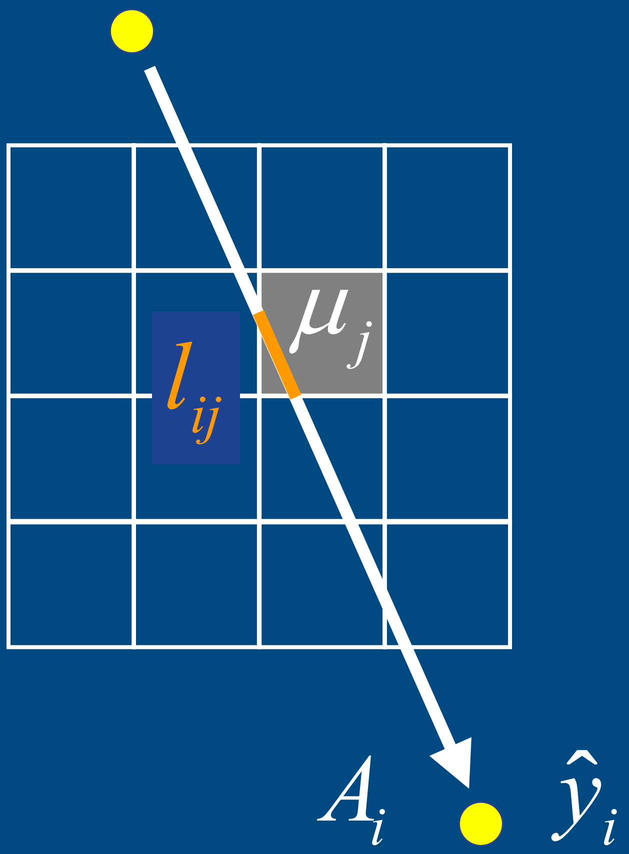
Basic CT forward model

$$\hat{y}_i = A_i \exp\left(-\sum_{j=1}^{J} l_{ij} \mu_j \right)$$



A_i: intensity sinogram in air

$\hat{y}_i$: calculated intensity sino

i : projection line (or sino) index

μ_j : linear attenuation coeff (1/cm)

j : image index

l_{ij} : intersection length (cm)

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Noise model

POISSON :

$$P(y_i \mid \hat{y}_i) = \frac{\hat{y}_i^{\,y_i} e^{-\hat{y}_i}}{y_i!}$$

$$\log P(y_i \mid \hat{y}_i) = y_i \log \hat{y}_i - \hat{y}_i - \log(y_i!)$$

GAUSSIAN :

$$P(y_i \mid \hat{y}_i) = \frac{1}{2\pi\sigma_i} e^{-\frac{(y_i - \hat{y}_i)^2}{2\sigma_i^2}}$$

$$\log P(y_i \mid \hat{y}_i) = -\frac{(y_i - \hat{y}_i)^2}{2\sigma_i^2} - \log(2\pi\sigma_i)$$

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Likelihood

Forward model

$$\hat{y}_i = A_i \exp\left(-\sum_{j=1}^{J} l_{ij}\mu_j\right)$$

Noise model

$$\log P(y_i \mid \hat{y}_i) = y_i \log \hat{y}_i - \hat{y}_i - \log(y_i!)$$

Likelihood

$$\log P(meas \mid calc) = \sum_i \left(y_i \log \hat{y}_i - \hat{y}_i\right)$$

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Cost functions

ML/MAP :

$$\arg\max_{\mu_j}\left[\overbrace{\sum_{i\in I}\left(y_i \ln \hat{y}_i - \hat{y}_i\right)}^{\text{LIKELIHOOD}} \overbrace{- \beta \sum_{j\in J}\sum_{k\in J} N_{jk}\phi\left(\mu_j - \mu_k\right)}^{\text{PRIOR}}\right]$$

WLS/PWLS :

$$\arg\min_{\mu_j}\left[\overbrace{\sum_{i\in I}\frac{1}{\sigma_i^2}\left(p_i - \hat{p}_i\right)^2}^{\text{DATAFIT}} + \overbrace{\beta \sum_{j\in J}\sum_{k\in J} N_{jk}\phi\left(\mu_j - \mu_k\right)}^{\text{REGULARIZER}}\right]$$

- y_i : measured intensity sino
- $\hat{y}_i$: calculated intensity sino
- p_i : measured int./atten. sino
- $\hat{p}_i$: calculated int./atten. sino
- β : prior weight
- N_{jk} : neighbourhood mask
- ϕ : potential function
- σ_i : standard deviation

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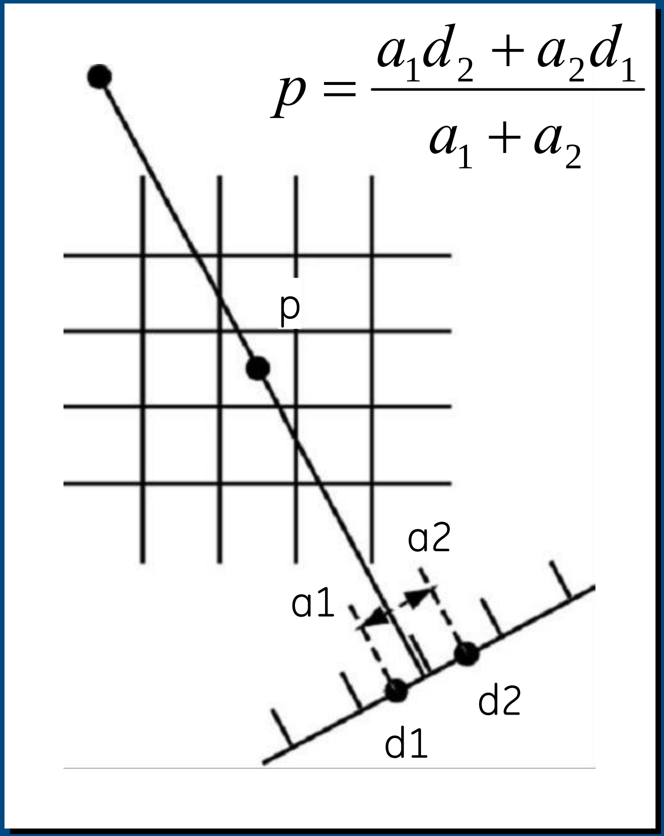
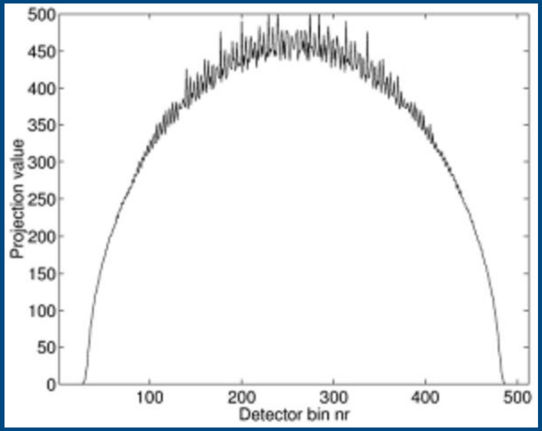
Pixel-driven with linear interpolation

Backprojection



Reprojection of uniform disk



Backprojection of uniform view

T. Peters, IEEE Trans. Nucl. Sci., 1981

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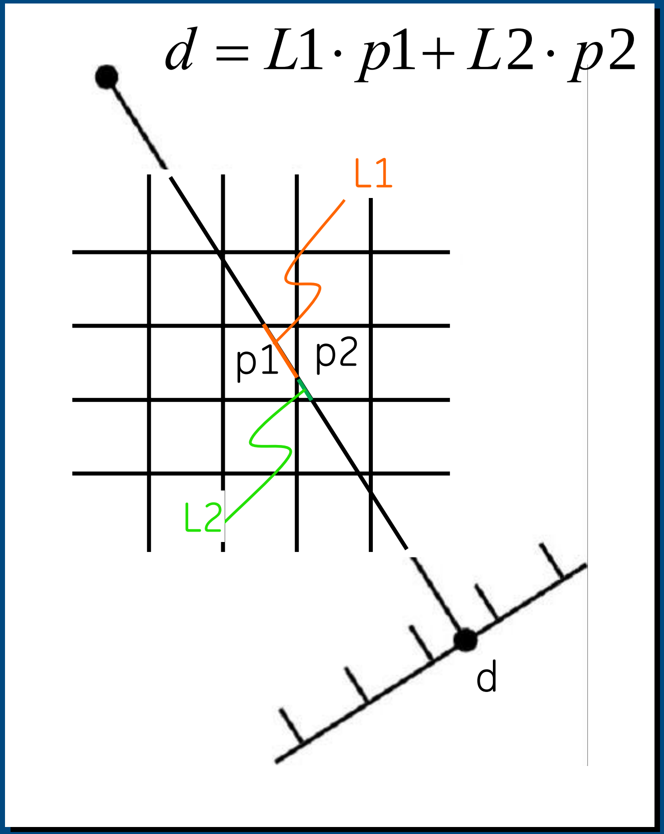
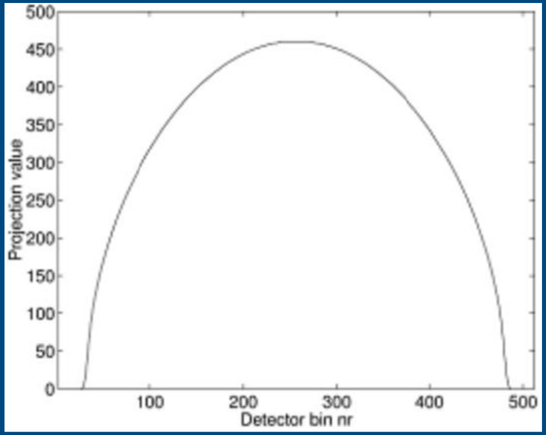
Ray-driven with intersection length

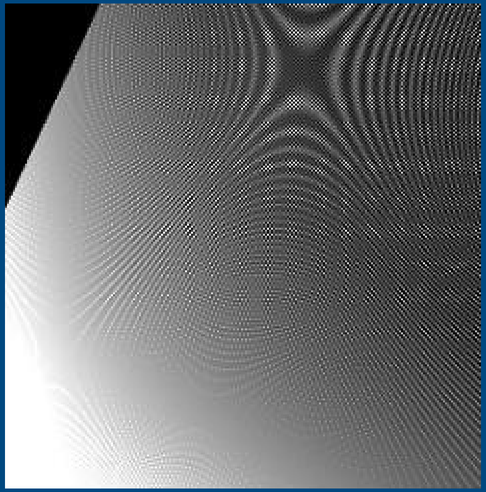
Re-projection



Reprojection of uniform disk



Backprojection of uniform view



R.Siddon, Med. Phys., 1985

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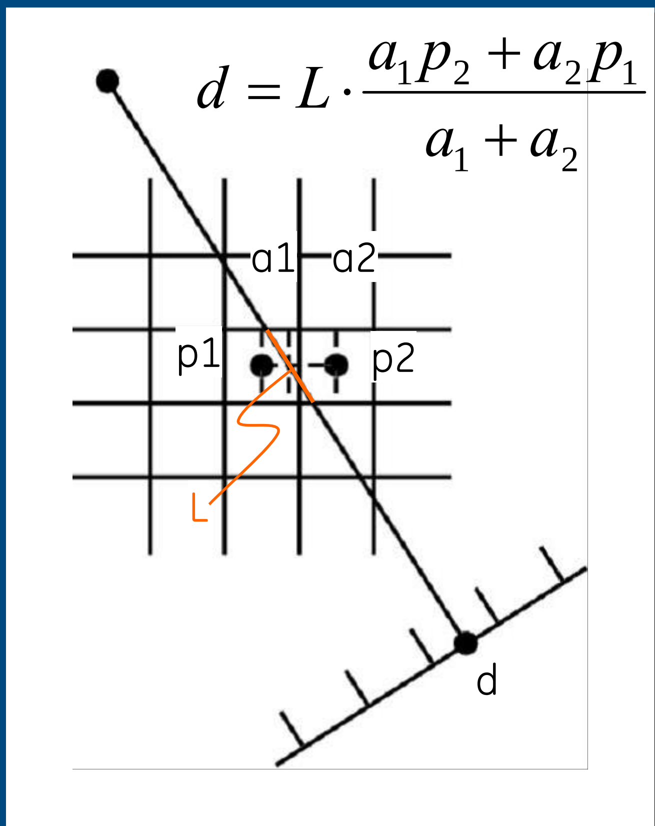
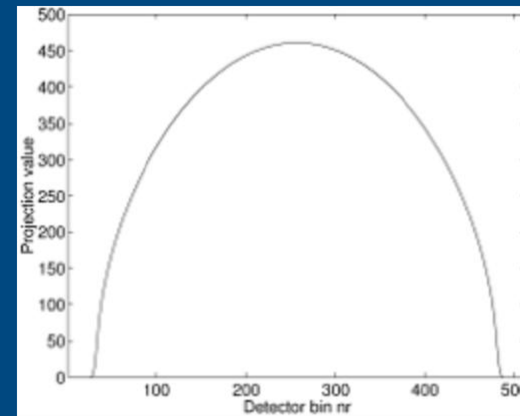
Ray-driven with linear interpolation

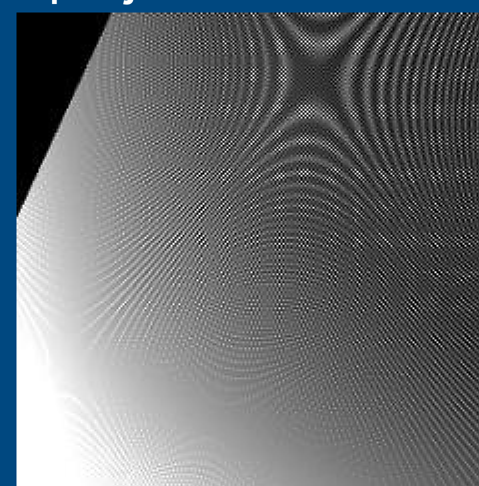
Re-projection



Reprojection of uniform disk



Backprojection of uniform view



P.M.Joseph, IEEE Trans. Med. Im., 1983

Bruno De Man
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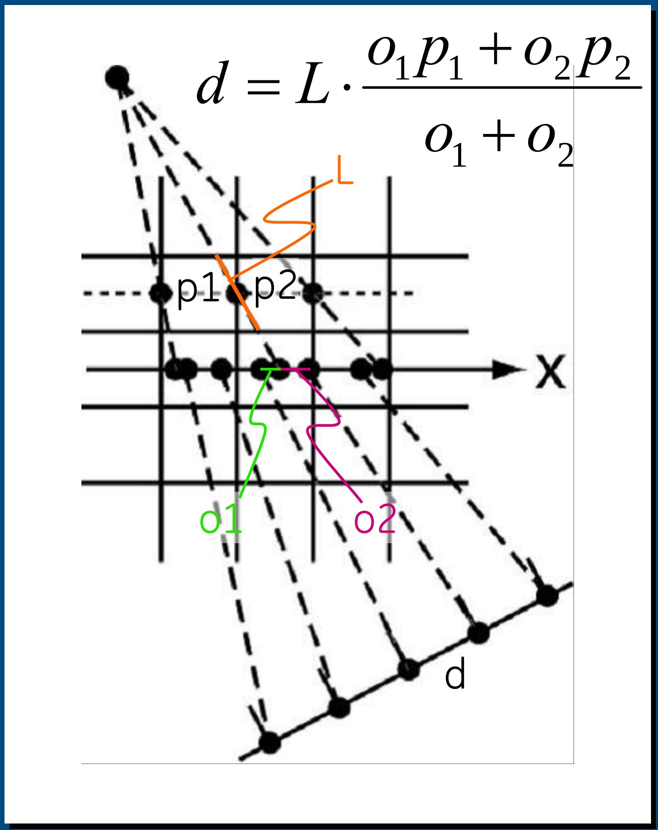
IEEE MIC, Honolulu, 2007

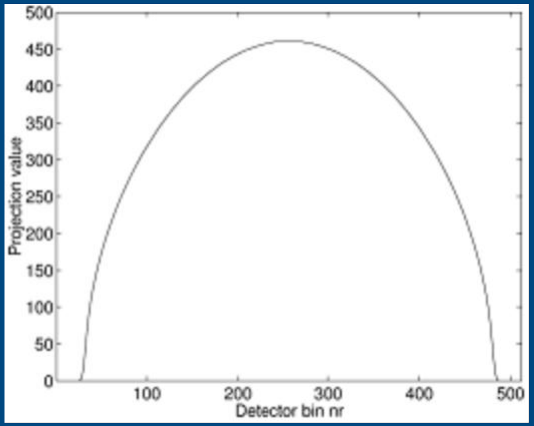
Distance-driven

Re-projection

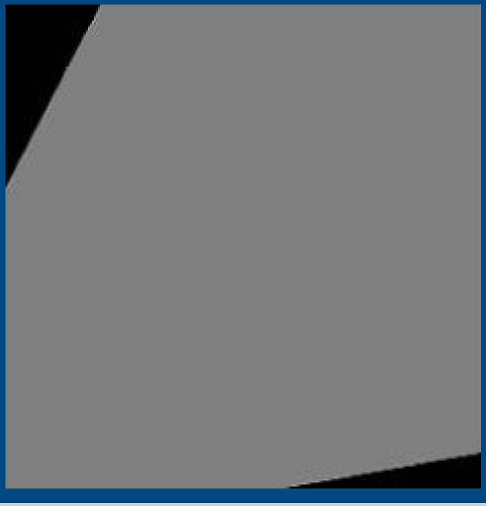


Based on pixel and detector *boundaries* !

Reprojection of uniform disk



Backprojection of uniform view



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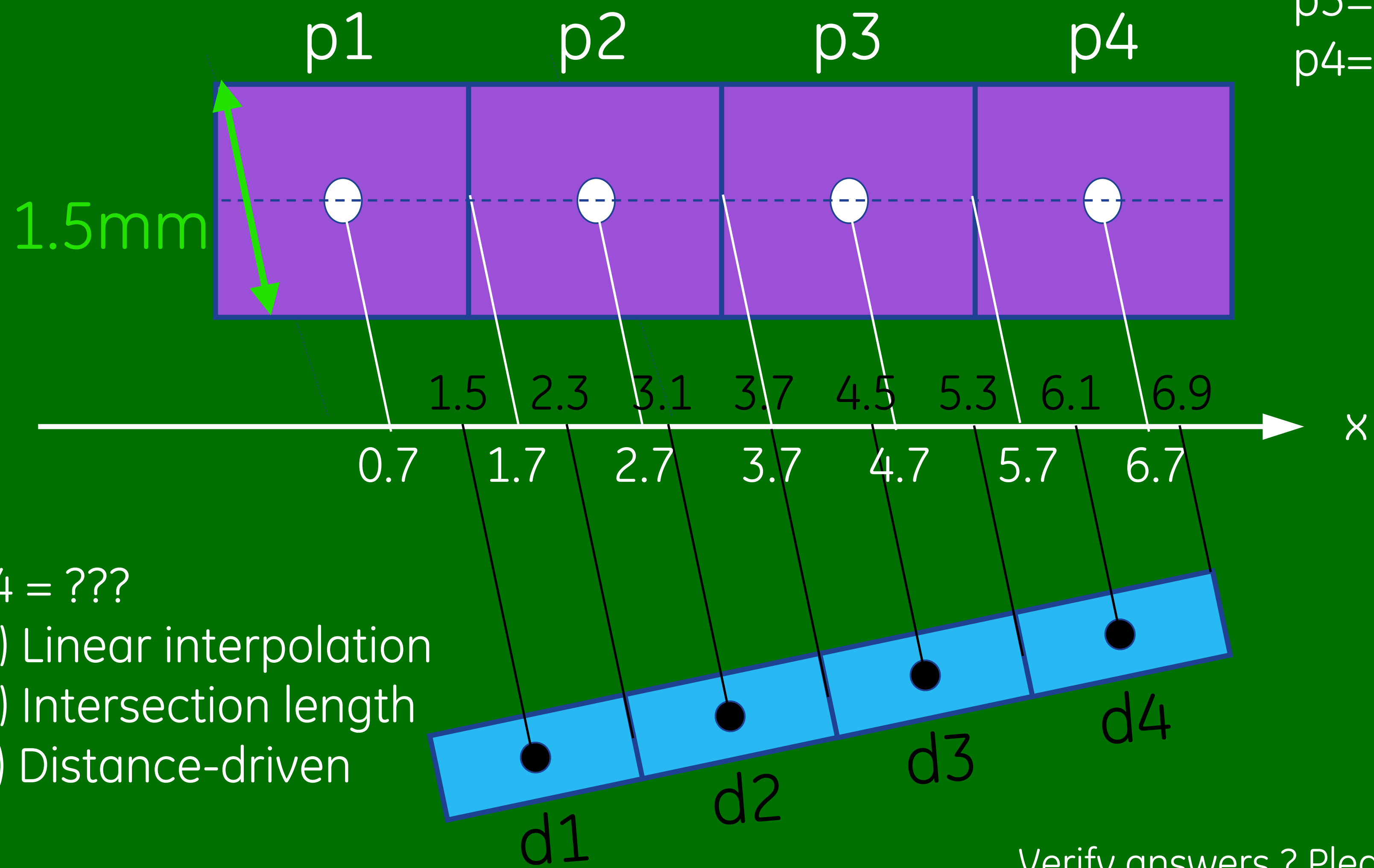
IEEE MIC, Honolulu, 2007

B. De Man, IEEE MIC, 2002

Exercise 2

$p1=0.2cm^{-1}$
$p2=0.4cm^{-1}$
$p3=0.6cm^{-1}$
$p4=0.4cm^{-1}$



d4 = ???

(a) Linear interpolation

(b) Intersection length

(c) Distance-driven

Verify answers ? Please email.

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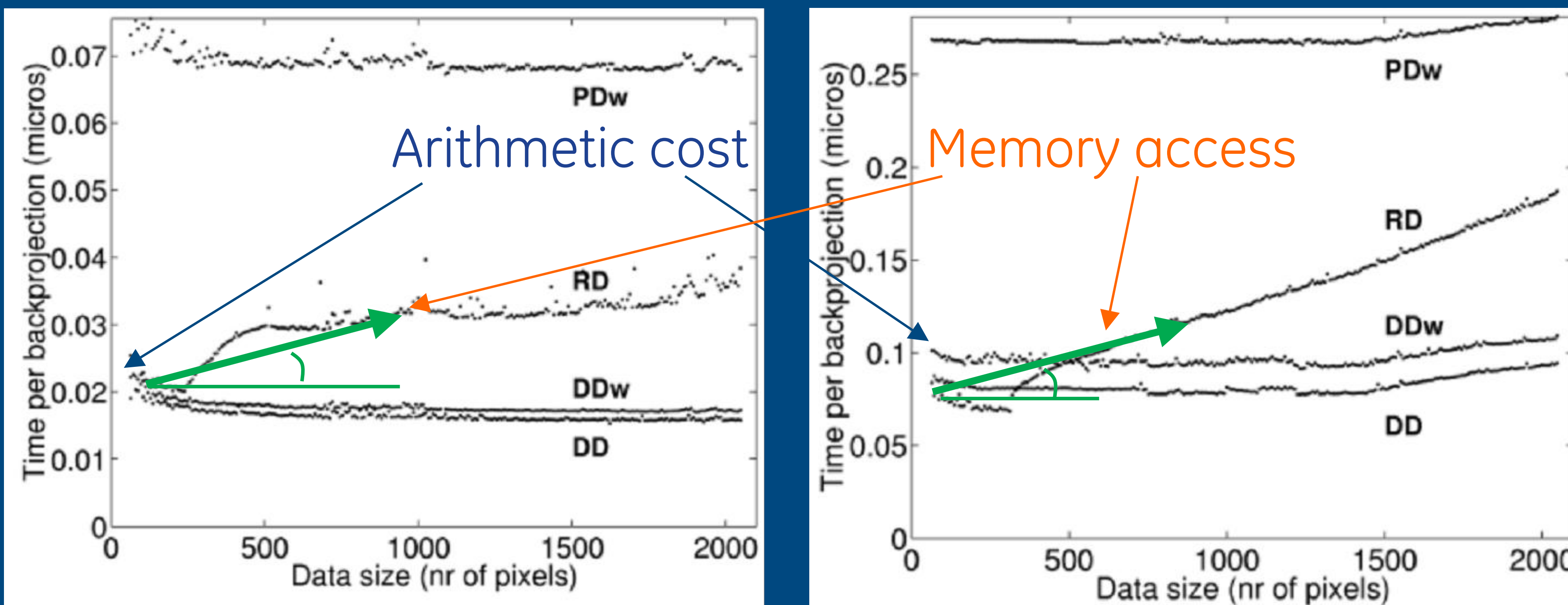
Computational performance

Pentium4 – 1.8Ghz **SUN E4500 UltraSparc-II**



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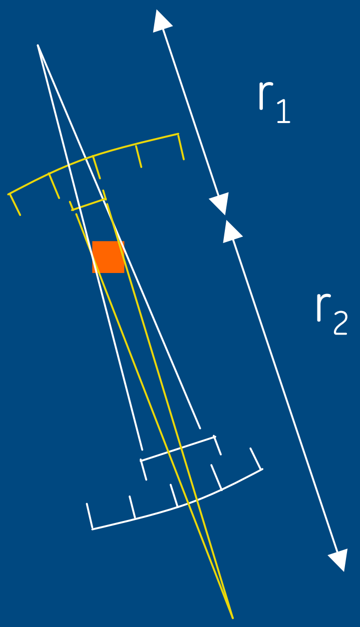
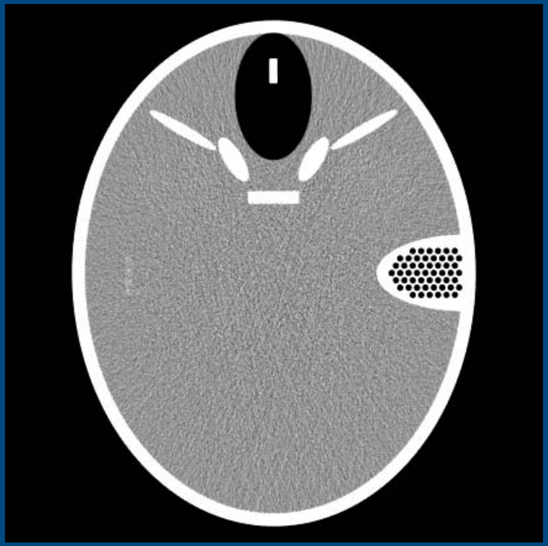
Distance-driven for FBP



512x512x16 FDK

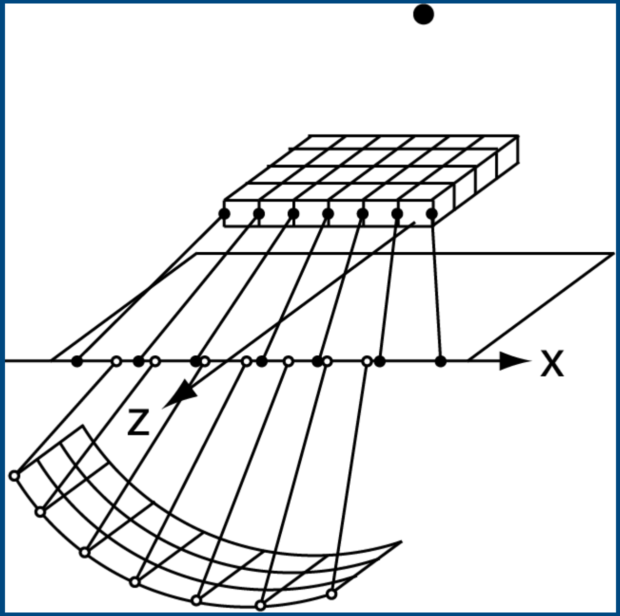
ROI	$FWHM_{PD}$	$FWHM_{DD}$	$\Delta\%$	σ^{PD}_{adj}	σ^{DD}_{adj}	$\Delta\%$
0	1.134	1.172	3.3	0.0309	0.0249	-19.5
1	1.029	1.051	2.2	0.0350	0.0323	-7.7
2	0.996	1.007	1.1	0.0350	0.0344	-1.8
3	1.021	1.014	-0.7	0.0315	0.0315	-0.1
4	1.059	1.060	0.1	0.0167	0.0168	0.0
5	1.011	1.022	1.1	0.0247	0.0257	4.1
6	0.905	0.898	-0.7	0.0391	0.0396	1.2



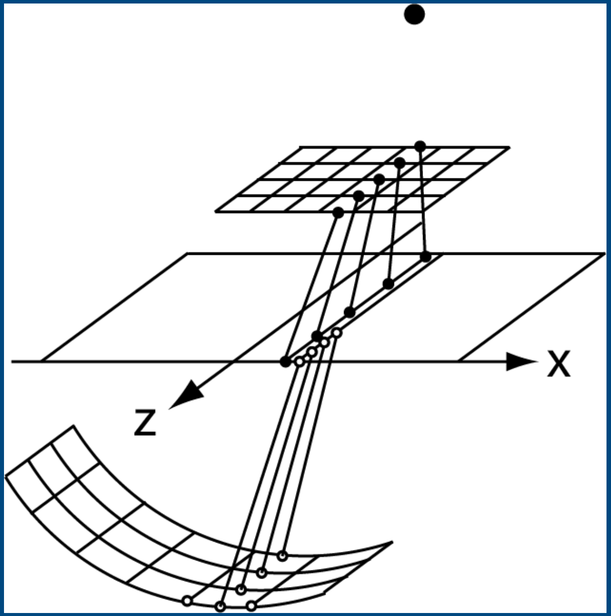
B. De Man, IEEE MIC, 2004

Distance-driven in 3D

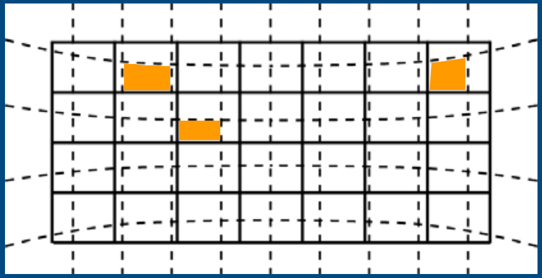
x-resampling



z-resampling



xz-overlap



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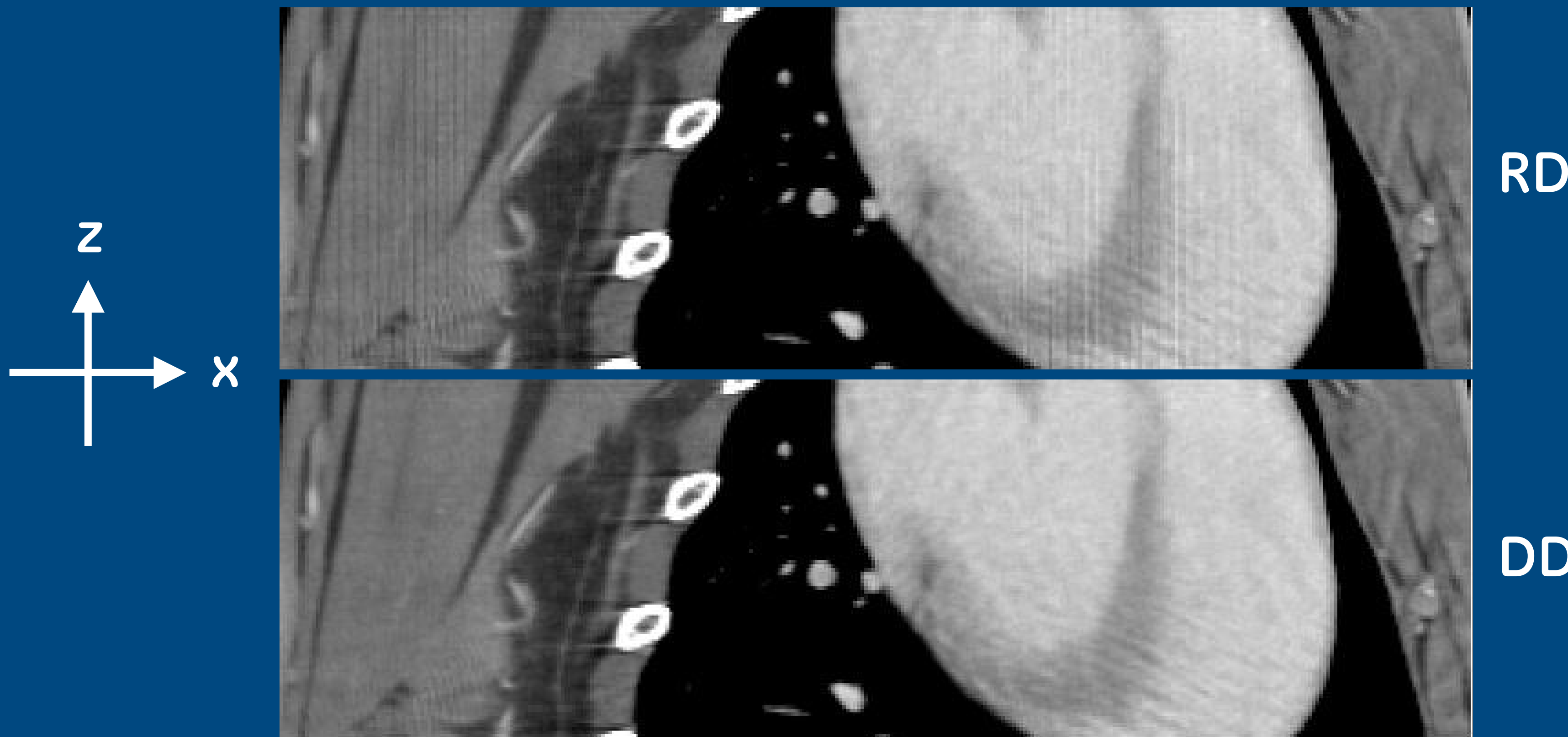
GE Global Research
IEEE MIC, Honolulu, 2007

B. De Man, Phys. Med. Biol., 2004

3D iterative recon example
(longitudinal reformat)



Bruno De Man
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MRF Gibbs priors

$$\log P(\mathrm{img}) = -\beta \sum_{j \in J} \sum_{k \in J} N_{jk} \phi(\mu_j - \mu_k)$$

β : prior weight

j, k : image index

J : entire image

μ_j : attenuation coeff

0.71	1.00	0.71
1.00	0.00	1.00
0.71	1.00	0.71

N_{jk} : neighbourhood mask

ϕ : potential function (determines prior type)

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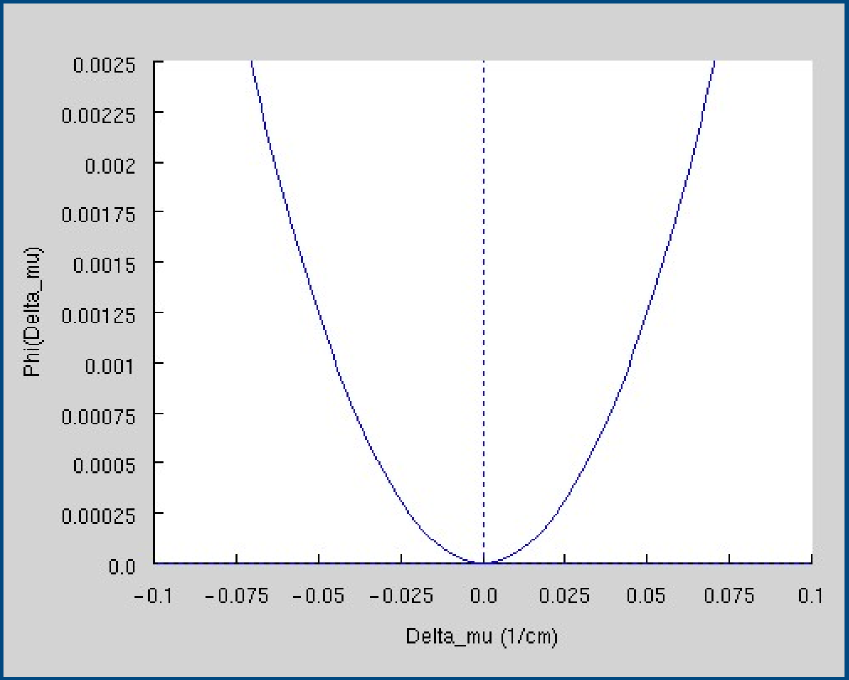
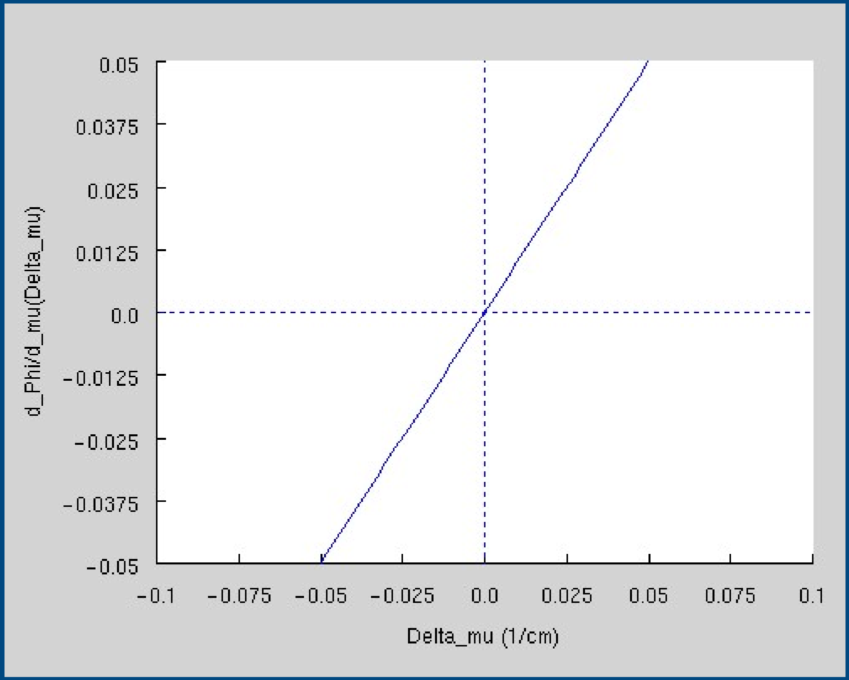
Quadratic prior

$$\phi(\mu) = \frac{\mu^2}{2}$$

$$\phi'(\mu) = \mu$$





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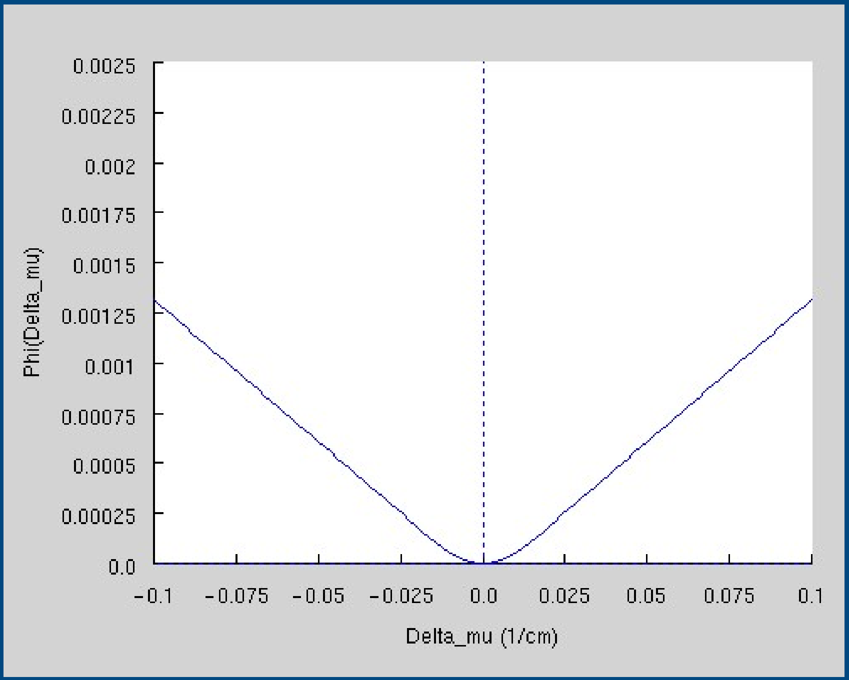
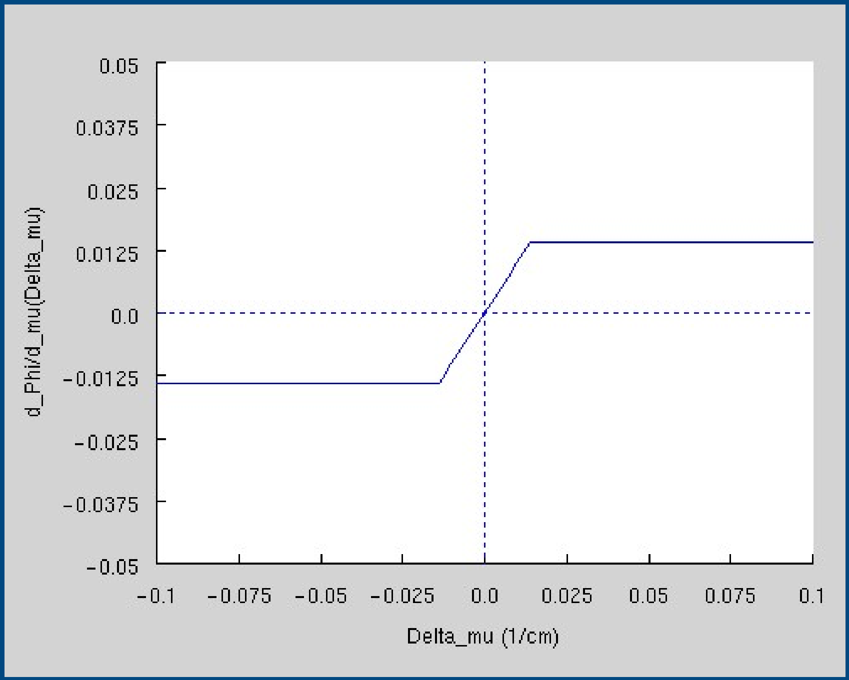
E. Mumcuoglu; Phys. Med. Biol., 1996

Huber prior

$$\phi(\mu) = \begin{cases} \dfrac{\mu^2}{2} & \text{for } |\mu| \le \delta \\ \delta\left(|\mu| - \delta/2\right) & \text{for } |\mu| > \delta \end{cases}$$

$$\phi'(\mu) = \begin{cases} \mu & \text{for } |\mu| \le \delta \\ \delta & \text{for } |\mu| > \delta \end{cases}$$



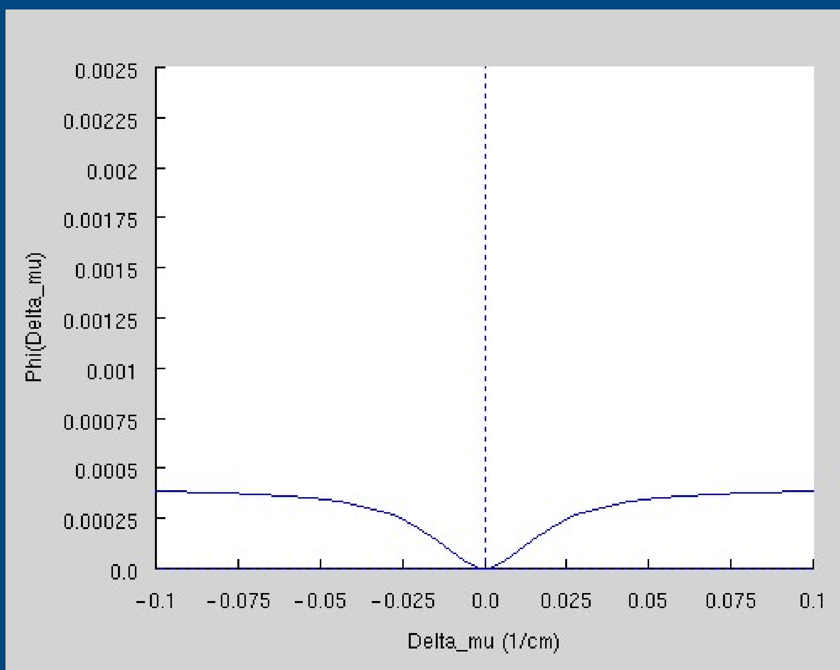


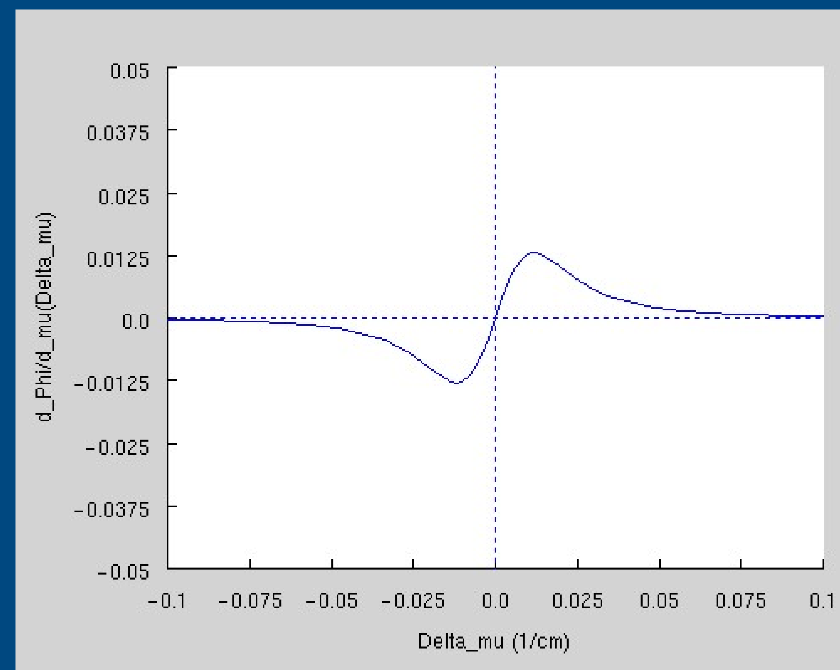
E. Mumcuoglu; Phys. Med. Biol., 1996

Geman prior

$$\phi(\mu) = \frac{\mu^2 \delta^2}{2\left(\mu^2 + \delta^2\right)}$$

$$\phi'(\mu) = \frac{\mu \delta^4}{\left(\mu^2 + \delta^2\right)^2}$$





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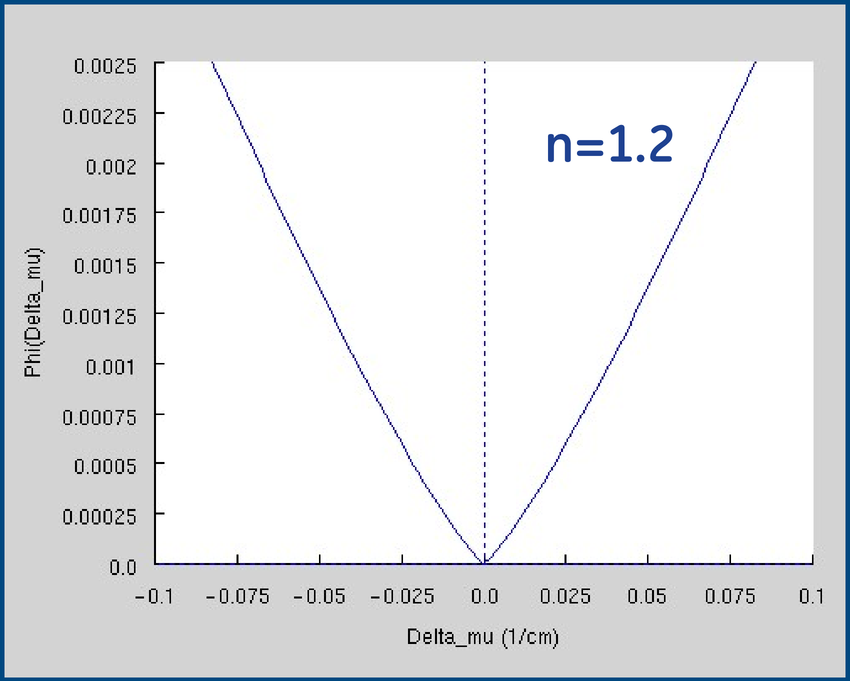
IEEE MIC, Honolulu, 2007

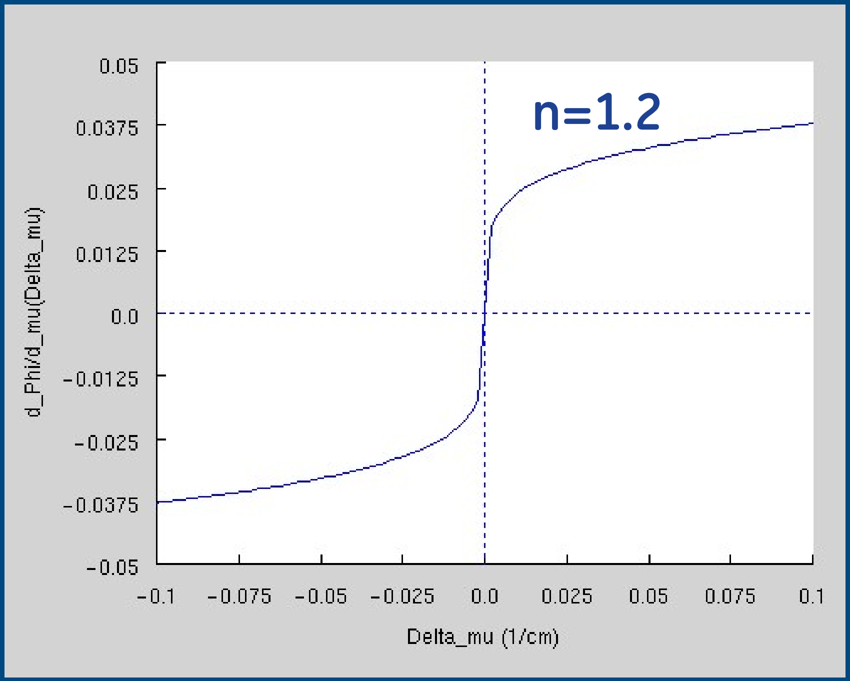
S Geman, Proc. Statist. Comput. Sect. Amer. Stat. Assoc., 1985

Generalized Gaussian prior

$$\phi(\mu) = \frac{|\mu|^n}{2}$$

$$\phi'(\mu) = \frac{n \cdot \mathrm{sign}(\mu) \cdot |\mu|^{n-1}}{2}$$





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IEEE MIC, Honolulu, 2007

C. Bouman, IEEE Trans. Img. Proc., 1993

Generalized Geman prior

$$\phi(\mu) = \frac{\mu^2 \delta^2}{2\left(\mu^2 + \delta^2\right)}$$

$$\phi'(\mu) = \frac{\mu \delta^4}{\left(\mu^2 + \delta^2\right)^2}$$

B. De Man, S. Basu: Generalized Geman prior for iterative reconstruction. In: *14th International Conference of Medical Physics*, Sep 14-17, 2005

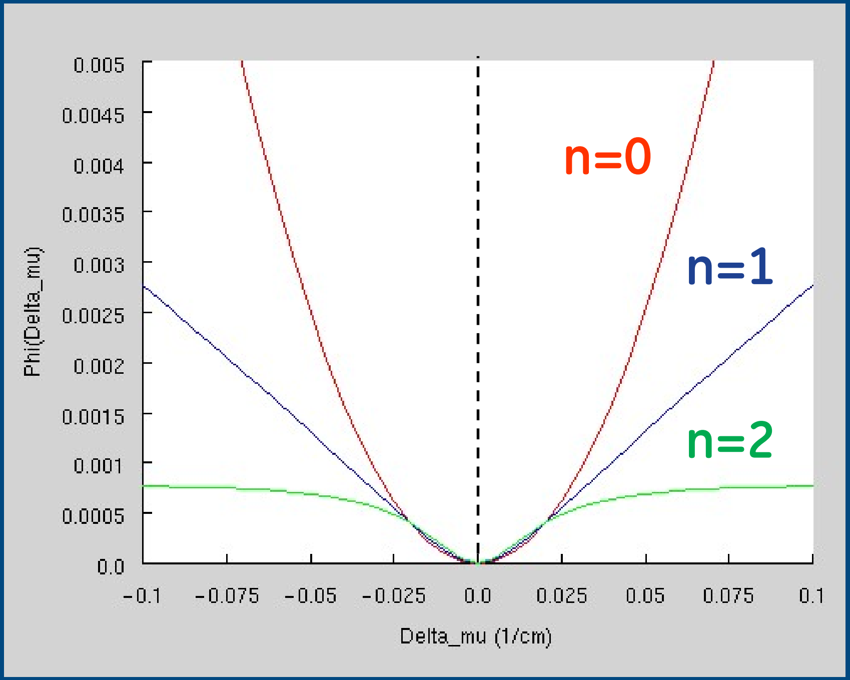
$$\phi(\mu) = \frac{\mu^2 \delta^n}{2\left(\sqrt{\mu^2/2 + \delta^2/2}\right)^n}$$

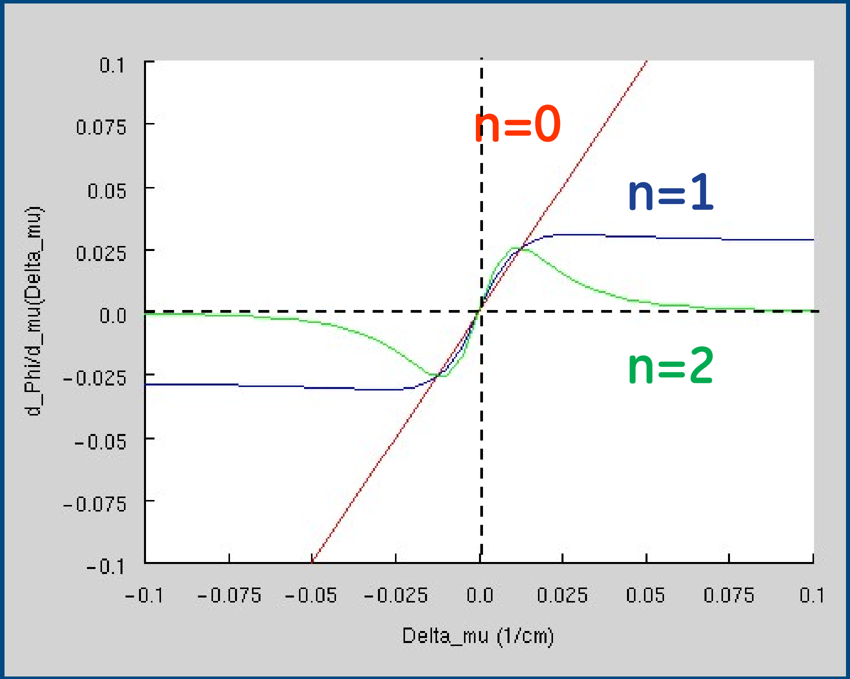
$$\phi'(\mu) = \frac{\mu \delta^n \left(\mu^2 (1 - n/2) + \delta^2\right)}{\left(\mu^2 + \delta^2\right)^{(n/2+1)}}$$

Generalized Geman prior

$$\phi(\mu) = \frac{\mu^2 \delta^n}{2\left(\sqrt{\mu^2/2 + \delta^2/2}\right)^n}$$

$$\phi'(\mu) = \frac{\mu \delta^n \left(\mu^2 (1 - n/2) + \delta^2\right)}{\left(\mu^2 + \delta^2\right)^{(n/2+1)}}$$





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Convexity condition

No local minima in MAP cost function
(necessary condition for convergence)

$\Uparrow$

Prior is strictly convex

$\Uparrow$

Second derivative ϕ'' is positive definite

$\Uparrow$

$n < 16/17$

$$\phi''(\mu) \propto \frac{\mu^4\left(n^2 - 3n + 2\right) + \mu^2\delta^2\left(4 - 5n\right) + 2\delta^4}{\left(\mu^2 + \delta^2\right)^{(n/2+2)}}$$

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Exercise 3

Derive the convexity condition for Generalized Geman prior :

Answer : $n < 16/17$

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Overview

- CT basics
- Noise models & cost functions
- Forward model – projector/backprojector
- Prior model
- → Update step
- Image quality
- Advanced forward model - incorporate physics

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Iterative Coordinate Descent

$$L = \frac{1}{2}\sum_i w_i \left(p_i - \hat{p}_i\right)^2 + \frac{1}{2}\sum_j \frac{1}{q\sigma^q} \sum_{k \in N_j} n(k-j) \frac{\left|\mu_j - \mu_k\right|^p}{1 + \left|\frac{\left(\mu_j - \mu_k\right)}{\delta}\right|^{p-q}}$$

$$\text{For n} = 1:\text{N} \qquad \text{(iteration number)}$$

$$\text{For j} = 1:\text{J} \qquad \text{(random scan pattern)}$$

$$\mu_j^n = \underset{\mu_j}{\arg\min}\left(L(\mu_1^n, \ldots, \mu_{j-1}^n, \mu_{j+1}^{n-1}, \ldots, \mu_J^{n-1})\right)$$

half-interval line search applied to find root of $\dfrac{\partial L}{\partial \mu}$

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K Sauer, IEEE Trans. Signal Proc., 1993

Exercise 4

ICD

$$L = \frac{1}{2}\sum_i w_i \left(p_i - \hat{p}_i^n\right)^2$$

$$\hat{p}_i^n = \sum_j l_{ij}\mu_j^n$$

$$j \neq k \rightarrow \mu_j^{n+1} = \mu_j^n$$

$$j = k \rightarrow \mu_j^{n+1} = \mu_j^n + \delta^{n+1}$$

Update step

$$\left.\frac{dL}{d\mu}\right|_{\mu_j^{n+1}} = 0 \Leftrightarrow \delta^{n+1} = ???$$

Verify answers ? Please email.

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MLTR / SPS derivation

1. The likelihood for transmission tomography

$$L(\mu) = \sum_i \ln p\left(y_i \mid b_i e^{-\sum_j l_{ij}\mu_j} + r_i \right)$$

$$= \sum_i \left(y_i \ln(b_i e^{-\sum_j l_{ij}\mu_j} + r_i) - (b_i e^{-\sum_j l_{ij}\mu_j} + r_i) \right)$$

$$= \sum_i h_i(\sum_j l_{ij}\mu_j)$$

with $h_i(x) = y_i \ln t_i(x) - t_i(x)$

$$t_i(x) = b_i e^{-x} + r_i$$

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J A Fessler, IEEE Trans. Med. Imag., 1997

< Courtesy of J. Nuyts (U. Leuven) >

MLTR / SPS derivation

2. Rewrite as a function of difference between new and old

$$\sum_j l_{ij}\mu_j = \sum_j \alpha_{ij}\left(\frac{l_{ij}}{\alpha_{ij}}\left(\mu_j - \mu_j^{\text{old}}\right) + \sum_k l_{ik}\mu_k^{\text{old}}\right)$$

with

$$\sum_j \alpha_{ij} = 1$$

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< Courtesy of J. Nuyts (U. Leuven) >

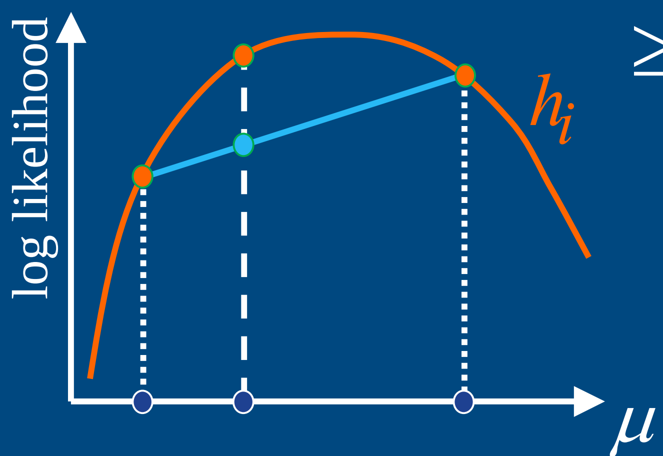
MLTR / SPS derivation

3. Use concavity

$$L(\mu) = \sum_i h_i\left(\sum_j l_{ij}\mu_j\right) = \sum_i h_i\left(\sum_j \alpha_{ij}\left(\frac{l_{ij}}{\alpha_{ij}}\left(\mu_j - \mu_j^{\text{old}}\right) + \sum_k l_{ik}\mu_k^{\text{old}}\right)\right)$$

$$\geq \sum_i \sum_j \alpha_{ij} h_i\left(\frac{l_{ij}}{\alpha_{ij}}\left(\mu_j - \mu_j^{\text{old}}\right) + \sum_k l_{ik}\mu_k^{\text{old}}\right)$$

$$= \sum_j Q_j\left(\mu_j, \mu^{\text{old}}\right)$$



with $Q_j\left(\mu_j, \mu^{\text{old}}\right) = \sum_i \alpha_{ij} h_i\left(\frac{l_{ij}}{\alpha_{ij}}\left(\mu_j - \mu_x^{\text{old}}\right) + \sum_k l_{ik}\mu_k^{\text{old}}\right)$

$$h_i(x) = y_i \ln t_i(x) - t_i(x)$$

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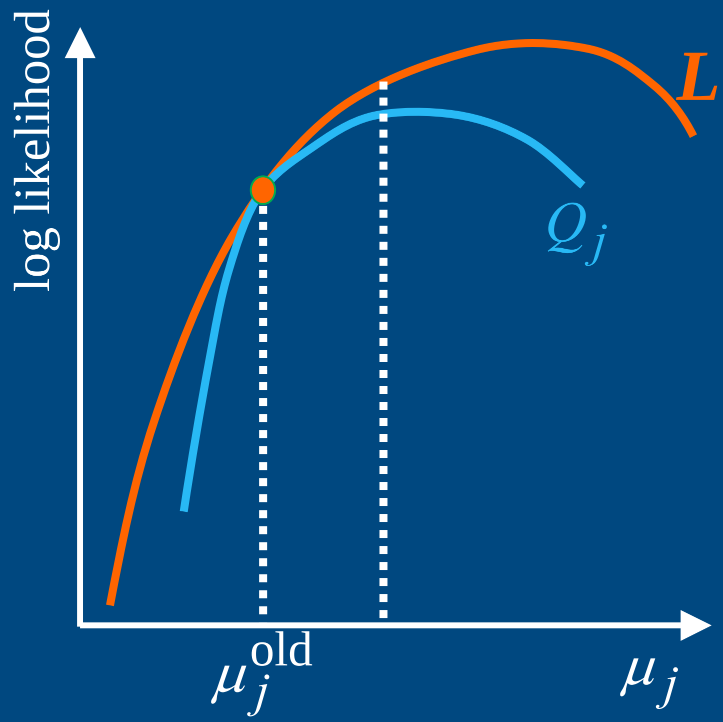
MLTR / SPS derivation

4. Maximizing (or increasing) *Q* increases *L*



$$L\left(\mu^{\text{old}}\right) = \sum_j Q_j\left(\mu_j^{\text{old}}, \mu^{\text{old}}\right)$$

$$L(\mu) \geq \sum_j Q_j\left(\mu_j, \mu^{\text{old}}\right)$$

$$\left.\frac{\partial L}{\partial \mu_j}\right|_{\mu_j^{\text{old}}} = \left.\frac{\partial Q_j}{\partial \mu_j}\right|_{\mu_j^{\text{old}}}$$

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< Courtesy of J. Nuyts (U. Leuven) >

MLTR / SPS derivation

5. Newton method

$$\mu_j = \mu_j^{\text{old}} + \Delta\mu_j \qquad \Delta\mu_j = \frac{\left.\dfrac{\partial Q_j}{\partial \mu_j}\right|_{\mu_j^{\text{old}}}}{-\left.\dfrac{\partial^2 Q_j}{\partial \mu_j{}^2}\right|_{\mu_j^{\text{old}}}}$$

$$\Delta\mu_j = \frac{\sum_i l_{ij} \dfrac{t_i - r_i}{t_i}(t_i - y_i)}{\sum_i \dfrac{l_{ij}}{\alpha_{ij}}(t_i - r_i)\dfrac{t_i^2 - r_i y_i}{t_i^2}}$$ **with** $$t_i = b_i e^{-\sum_k l_{ik}\mu_k^{\text{old}}} + r_i$$

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< Courtesy of J. Nuyts (U. Leuven) >

MLTR / SPS derivation

6. Choosing α_{ij}

Recall that $\sum_j \alpha_{ij} = 1$

Assume for simplicity that $r_i = 0$

$$\alpha_{ij} = \frac{l_{ij}\mu_j}{\sum_k l_{ik}\mu_k} \quad \Rightarrow \quad \Delta\mu_j = \frac{\mu_j \sum_i l_{ij}(t_i - y_i)}{\sum_i l_{ij}\left(\sum_k l_{ik}\mu_k\right)t_i}$$

$$\alpha_{ij} = \frac{l_{ij}}{\sum_k l_{ik}} \quad \Rightarrow \quad \Delta\mu_j = \frac{\sum_i l_{ij}t_i - \sum_i l_{ij}y_i}{\left(\sum_k l_{ik}\right)\sum_i l_{ij}t_i}$$

$$t_i = b_i e^{-\sum_k l_{ik}\mu_k} + r_i$$

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< Courtesy of J. Nuyts (U. Leuven) >

Exercise 5

$$L = \sum_i y_i \ln \hat{y}_i - \hat{y}_i$$

$$\hat{y}_i = A_i \exp(-\sum_j l_{ij} \mu_j)$$

$$\Delta\mu_j^{n+1} = -\frac{\left.\dfrac{\partial L}{\partial \mu_j}\right|_{\{\mu_j^n\}}}{\sum_{\xi \in J} \left.\dfrac{\partial^2 L}{\partial \mu_j \partial \mu_\xi}\right|_{\{\mu_j^n\}}}$$

$$\Delta\mu_j^{n+1} = ?$$

Verify answers ? Please email.

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J Nuyts, 2nd IEEE Workshop CMP, 1996
J Nuyts, Phys. Med. Biol., 1998

Ordered-subsets algorithms

C=Calculated
M=Measured

OS-MLTR (Nuyts '96)

$$\mu_j^{n+1} = \mu_j^n + \frac{\sum_{i \in S} l_{ij}(C_i - M_i)}{\sum_{i \in S} l_{ij} C_i \sum_{\xi \in J} l_{i\xi}}$$

OS-MLMOD

$$\mu_j^{n+1} = \mu_j^n + \frac{\sum_{i \in S} l_{ij}(1 - M_i / C_i) M_i}{\frac{|S|}{|I|} \sum_{i \in I} l_{ij} M_i \sum_{\xi \in J} l_{i\xi}}$$

OS-SPS (Fessler '97)

$$\mu_j^{n+1} = \mu_j^n + \frac{\sum_{i \in S} l_{ij}(C_i - M_i)}{\sum_{i \in S} l_{ij} M_i \sum_{\xi \in J} l_{i\xi}}$$

OS-WLS

$$\mu_j^{n+1} = \mu_j^n + \frac{\sum_{i \in S} l_{ij} \left(P_i - \sum_{k \in S} l_{ik} \mu_k^n \right) M_i}{\frac{|S|}{|I|} \sum_{i \in I} l_{ij} M_i \sum_{\xi \in J} l_{i\xi}}$$

J Nuyts, 2nd IEEE Workshop CMP, 1996
J A Fessler, IEEE Trans. Med. Imag., 1997
B. De Man, ICMP, 2005

Convergent Ordered Subsets

1. Express log-likelihood as sum of N subset terms
2. For each subset, update gradient for one term
3. Perform image update using most recent gradients

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Conjugate Gradients

$$L = \frac{1}{2}\sum_i w_i \left(p_i - \hat{p}_i\right)^2 + \sum_j \alpha \sum_{\substack{k \in N_j \\ k > j}} n(k-j) \frac{\left(\mu_j - \mu_k\right)^2}{2}$$

$$d^{n+1} = \nabla L + \gamma^n d^n$$

$$\widehat{\alpha} = \arg\min_{\alpha} L\left(x^n + \alpha d^{n+1}\right)$$

$$x^{n+1} = x^n + \widehat{\alpha} d^{n+1}$$

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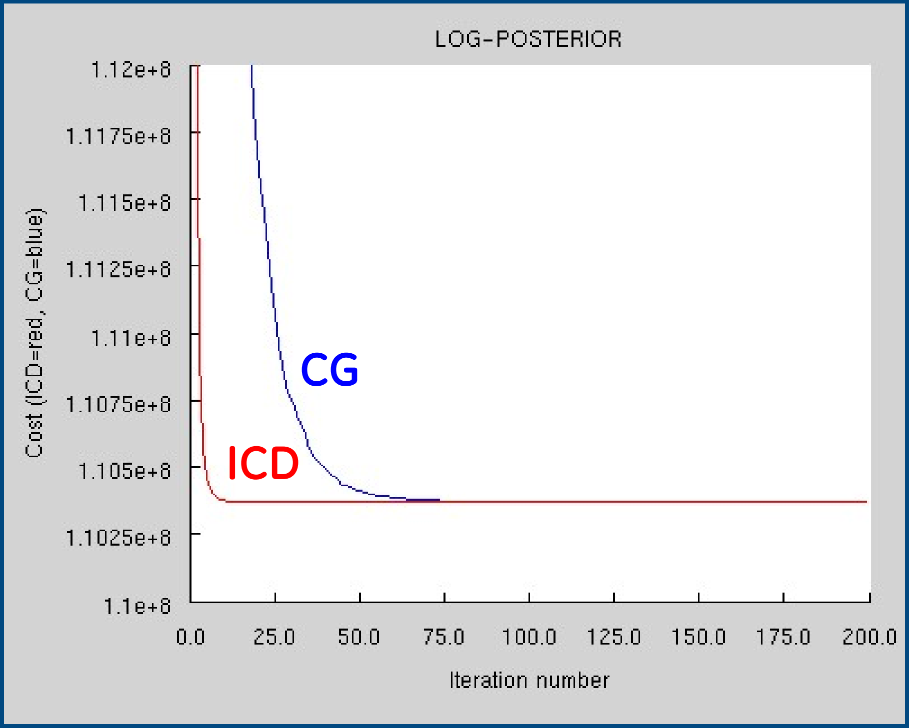
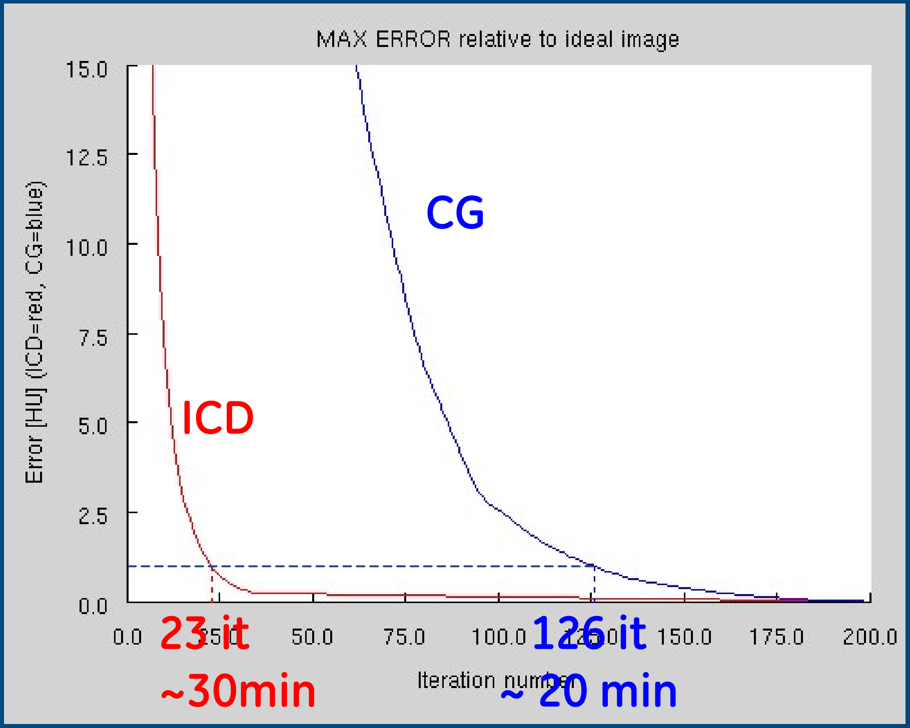
Results : ICD versus CG

No positivity-constraint
Initialize with FBP
200 iterations

ICD (200 iter)	
Lik	4.2992e+07
Prior	6.7379e+07
Post	1.1037e+08

CG (200 iter)	
Lik	4.2993e+07
Prior	6.7376e+07
Post	1.1037e+08





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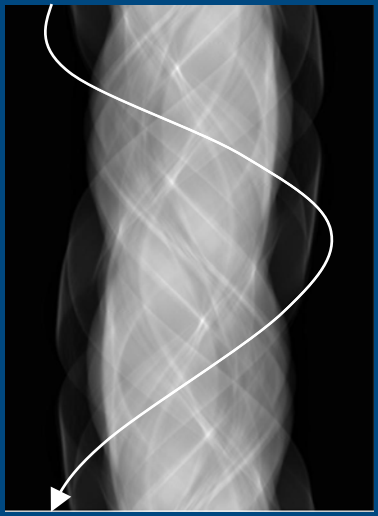
Performance tradeoff

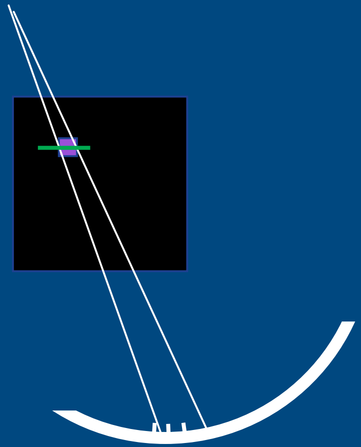
Simultaneous update :
- many iterations
- low arithmetic cost
- sequential memory access

Iterative coordinate descent :
- few iterations
- high arithmetic cost
- “random” memory access

Grouped-coordinate methods :
- combine the best of both worlds ?



ICD memory access pattern



DD projector for ICD

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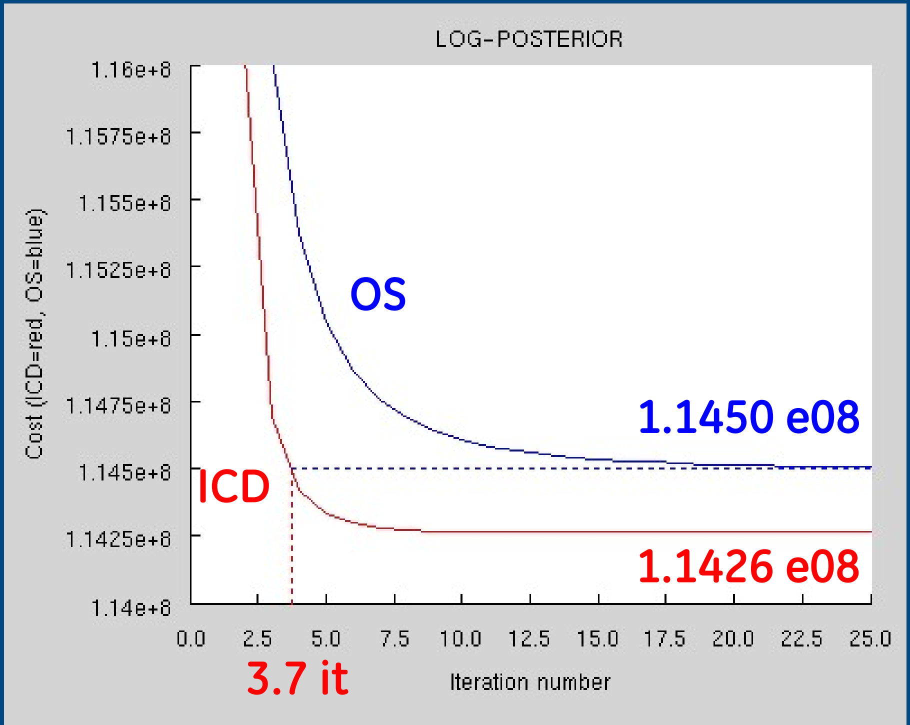
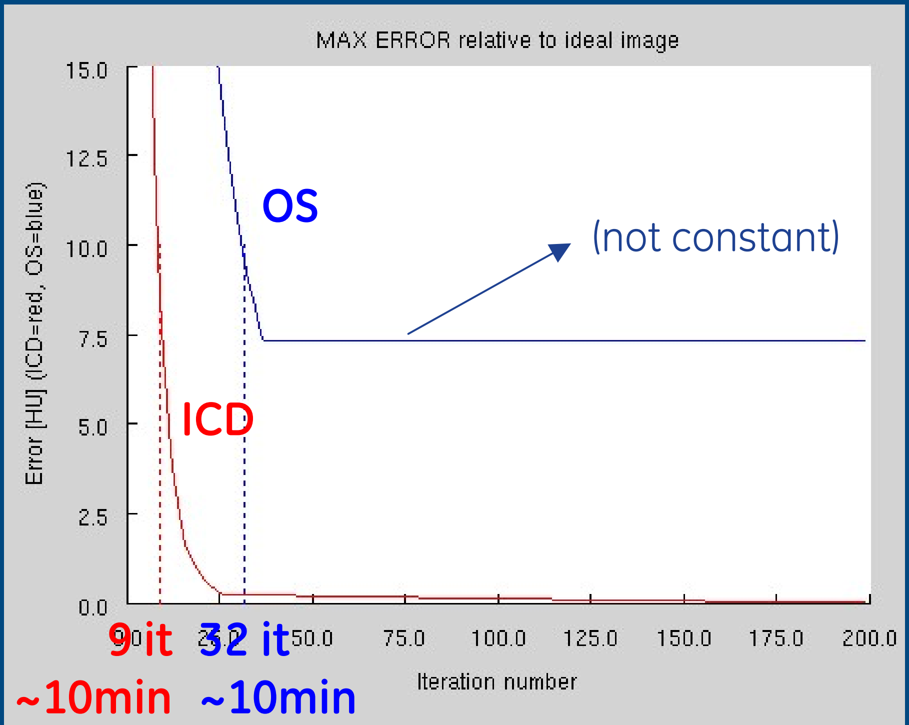
Results : ICD versus OS

With positivity-constraint

Initialize with FBP

200 iter
41 subsets (OS)





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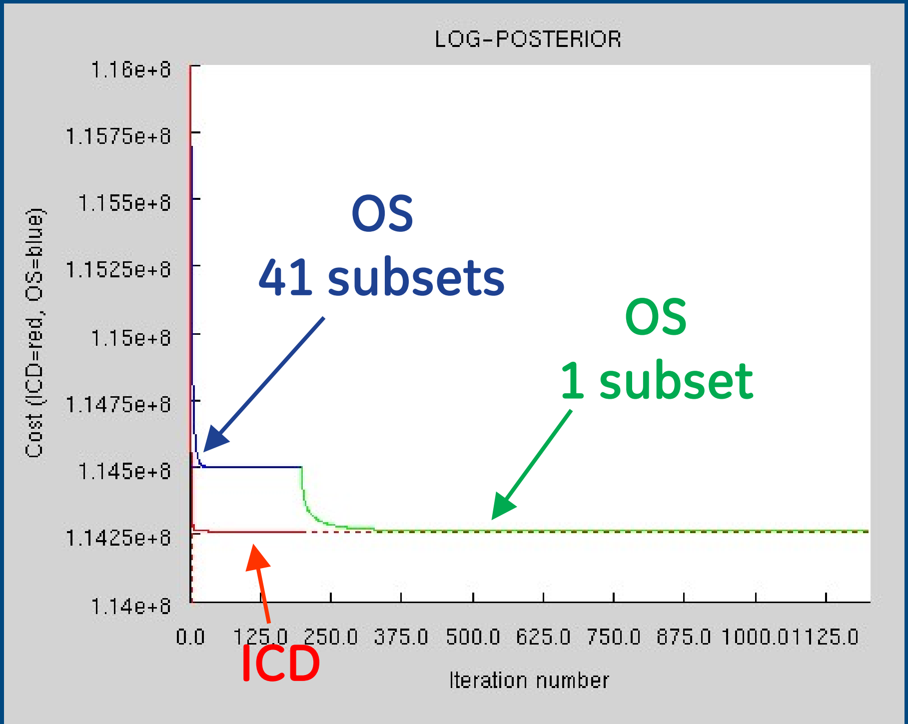
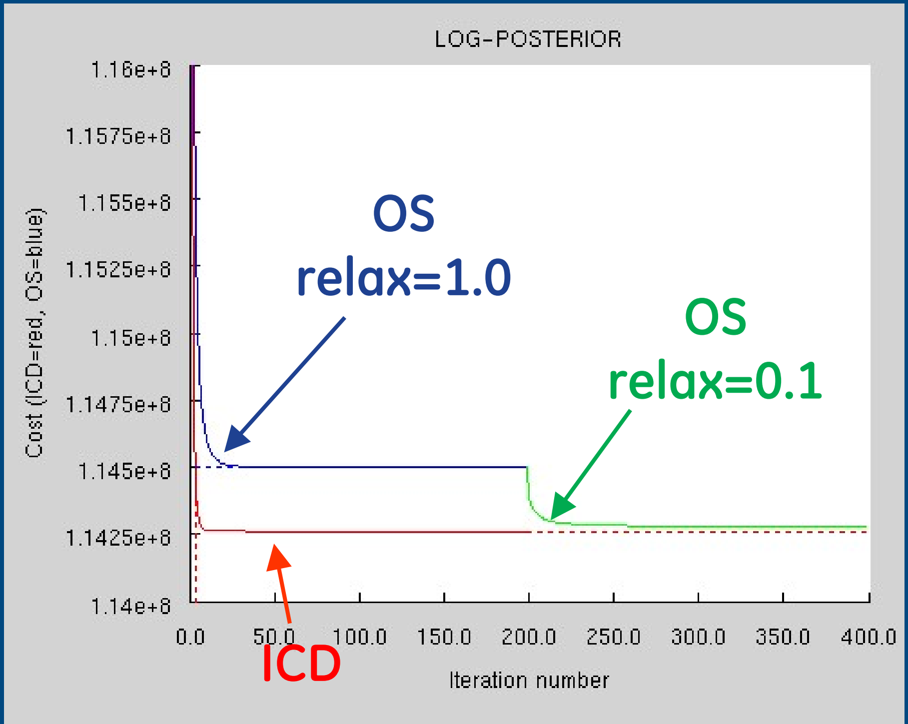
Results : ICD versus OS

With positivity-constraint

Initialize with FBP





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Results : ICD versus OS

With positivity-constraint

Initialize with FBP

200 iter
41 subsets (OS)

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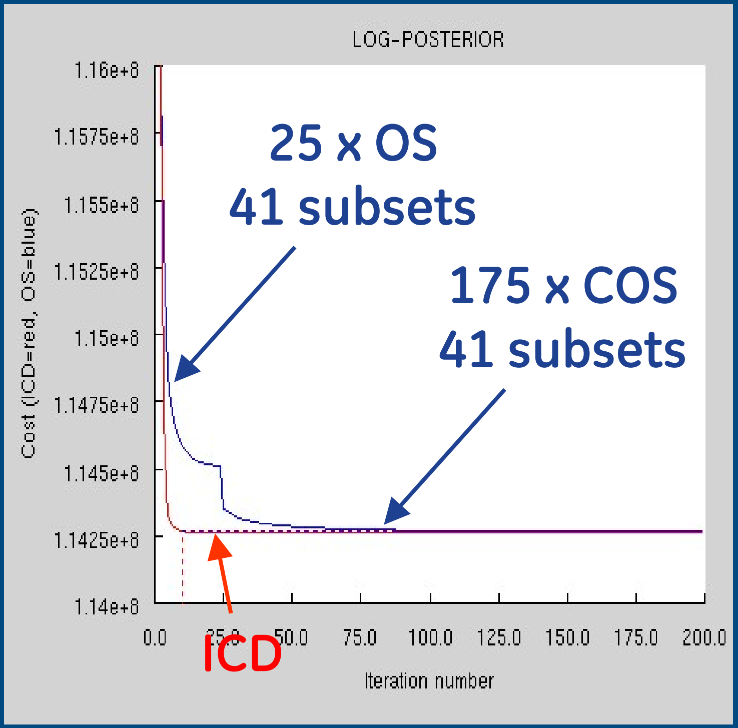
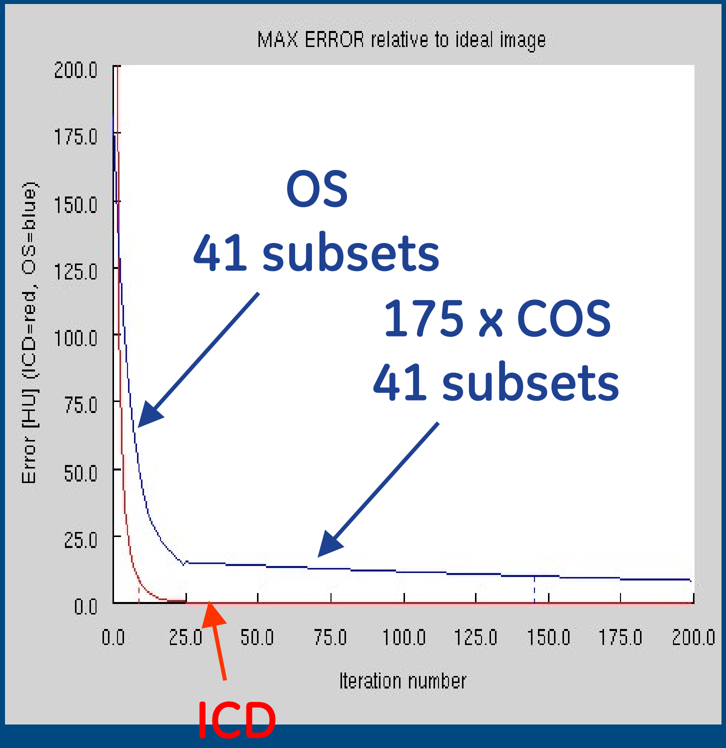
Results : ICD versus COS

With positivity-constraint

Initialize with FBP





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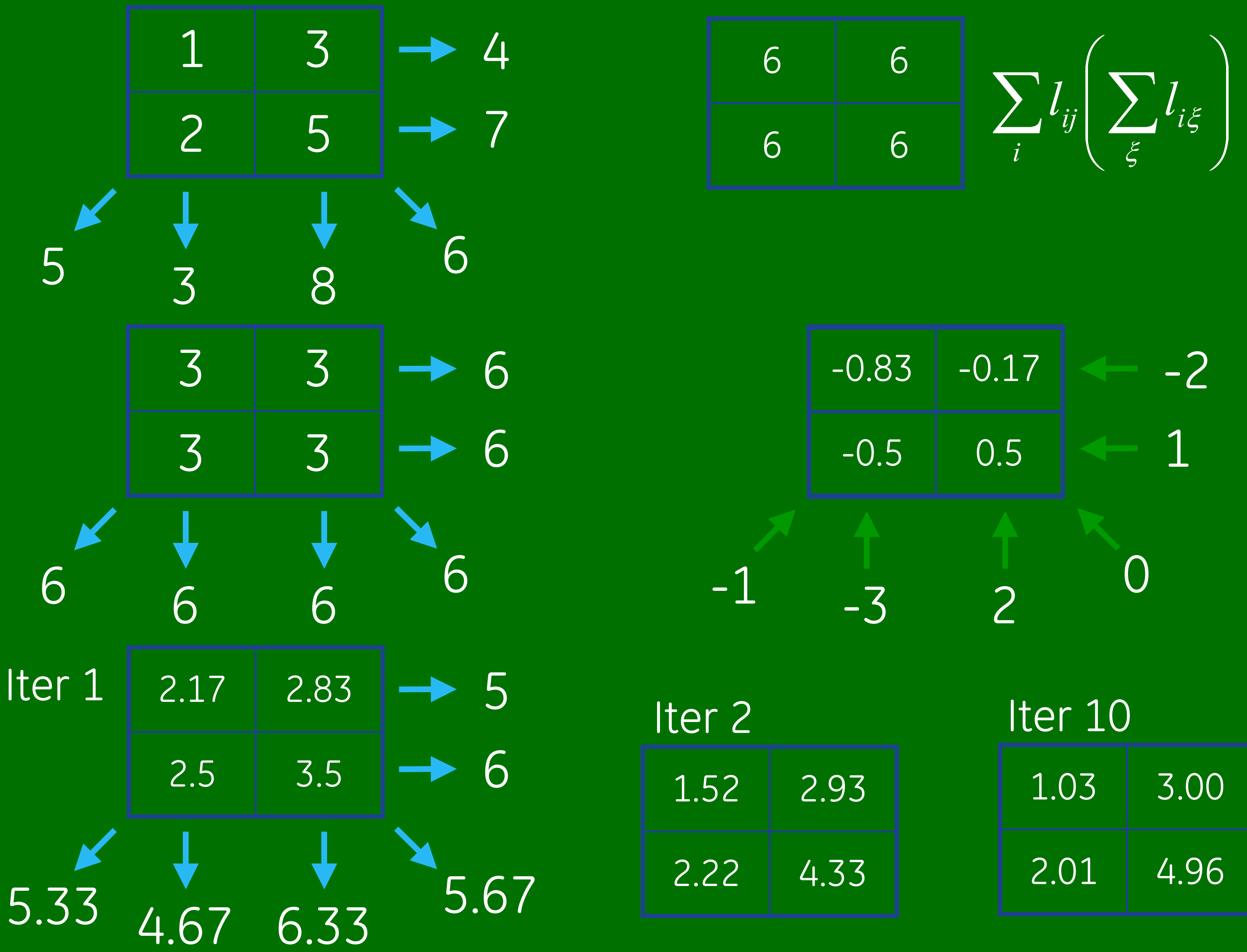
Exercise 6



1 3 → 4
2 5 → 7
5 3 8 6
3 3 → 6
3 3 → 6
6 6 6 6
Iter 1
2.17 2.83 → 5
2.5 3.5 → 6
5.33 4.67 6.33 5.67
6 6
6 6
$\sum_i l_{ij} \left(\sum_\xi l_{i\xi} \right)$
-0.83 -0.17 ← -2
-0.5 0.5 ← 1
-1 -3 2 0
Iter 2
1.52 2.93
2.22 4.33
Iter 10
1.03 3.00
2.01 4.96

Exercise 6

1	3	→ 4
2	5	→ 7

5 3 8 **7**

3	3	→ 6
3	3	→ 6

6 6 6 6

6	6
6	6

$$\sum_i l_{ij}\left(\sum_\xi l_{i\xi}\right)$$

-0.67	-0.17	← -2
-0.5	**0.67**	← 1

-1 -3 2 **1**

Iter 1

2.33	2.83
2.5	**3.67**

Iter 2

2	2.83
2.28	4.22

Iter 10

1.36	2.83
1.85	5.29

Exercise 6

1	3
2	5

4

7

5 3 8 **7** → w=0.1

4.2	6
6	4.2

$$\sum_i l_{ij} w_i \left(\sum_{\xi} l_{i\xi} \right)$$

3	3
3	3

6

6

6 6 6 6

-1.17	-0.17
-0.5	**0.74**

-2

1

-1 -3 2 **1**

Iter 1

1.83	2.83
2.5	**3.74**

Iter 2

1.39	2.90
2.35	4.29

Iter 10

1.04	2.98
2.00	5.04

Overview

- CT basics
- Noise models & cost functions
- Forward model – projector/backprojector
- Prior model
- Update step
- Image quality
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Iterative recon IQ properties

Dependent on voxel size :

- voxel size determines number of unknowns
- effect of prior depends on voxel size

Non-linearity :

- effect of prior depends on object contrast
- strongly attenuating objects impact statistics, and therefore noise and resolution

Spatial dependence :

- system geometry results in non-uniform/non-isotropic IQ
- local statistics determines noise, but also impacts resolution

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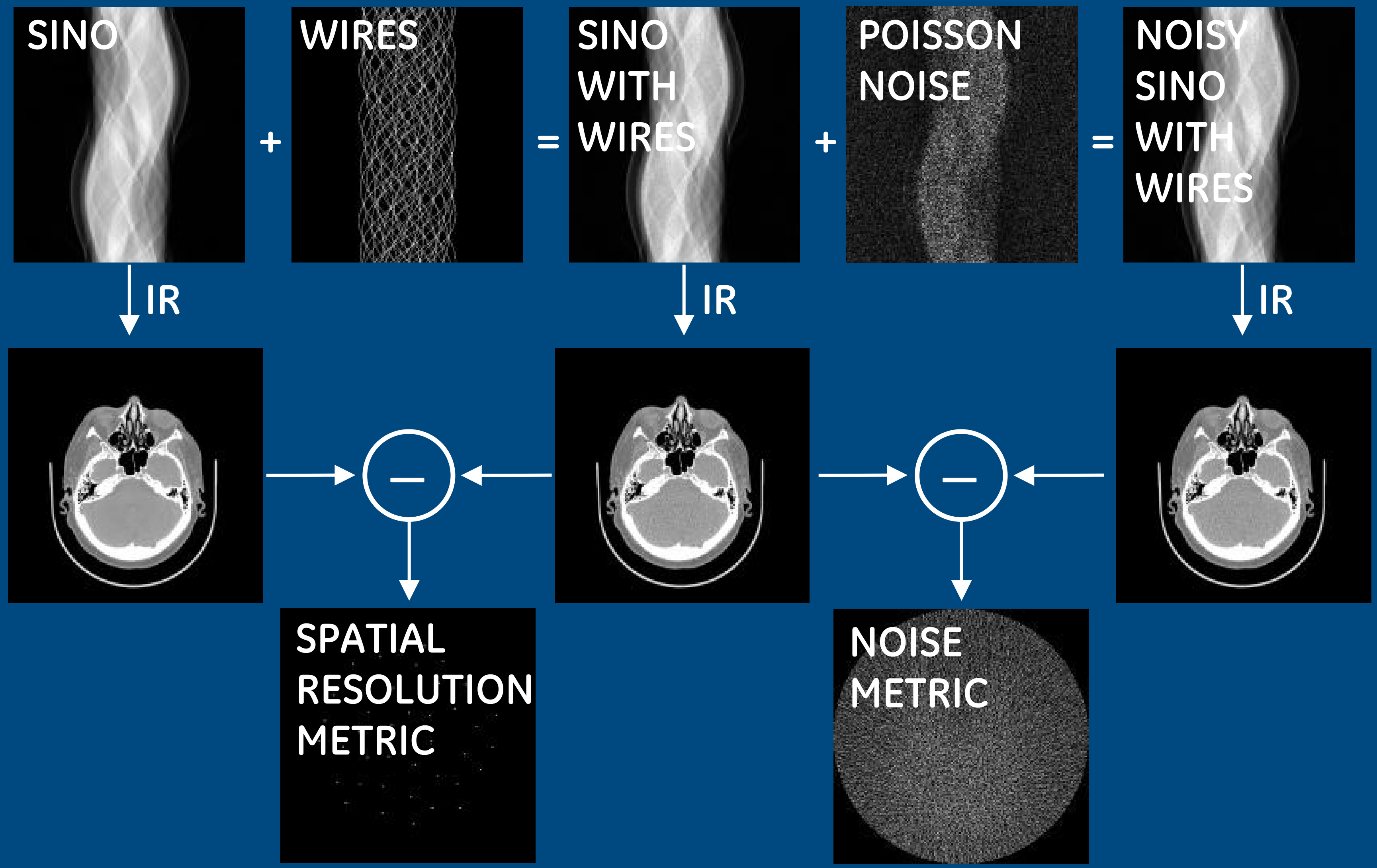
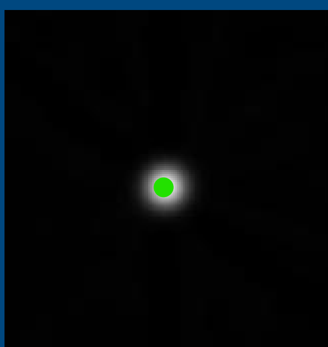
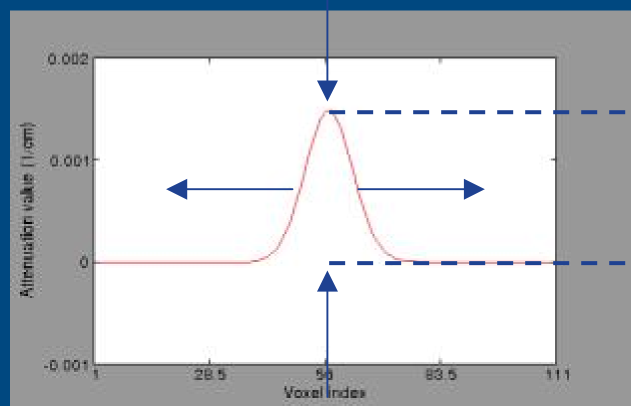
Noise-resolution metric



Use wire amplitude as a relative resolution metric

wire image → **wire profile** → **wire amplitude**





$$\sigma \sim \sqrt{\frac{\iint f}{\max(f)}}$$

$$\mathrm{PSF\_Integral}/\mathrm{PSF\_Max} = 2\pi(\sigma/\mathrm{pixelsize})^2$$

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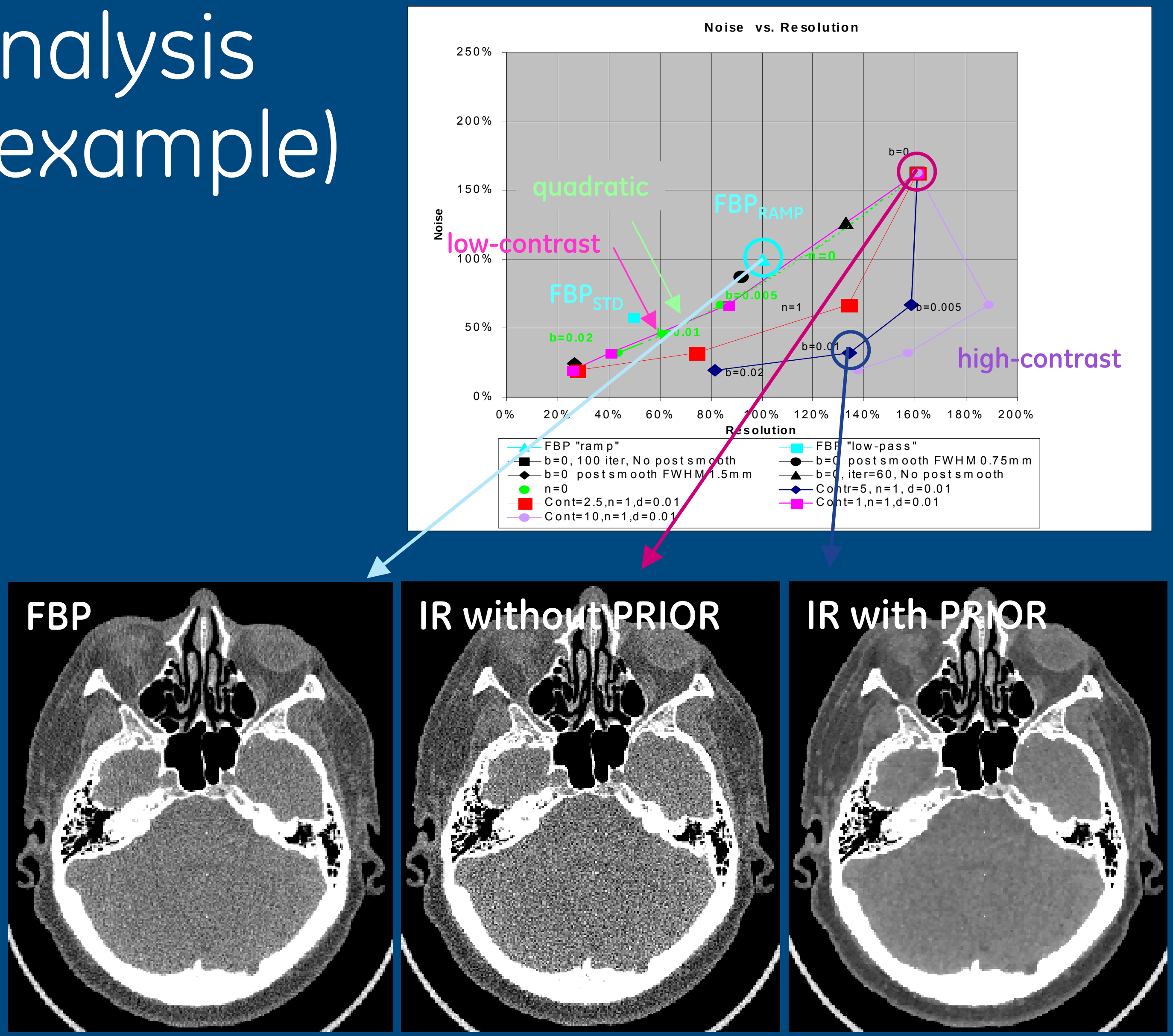
IEEE MIC, Honolulu, 2007

B. De Man, Imaging, 2006

IQ analysis (2D example)



M. Iatrou, IEEE Med. Im. Conf., 2006 & 2007

Overview

- CT basics
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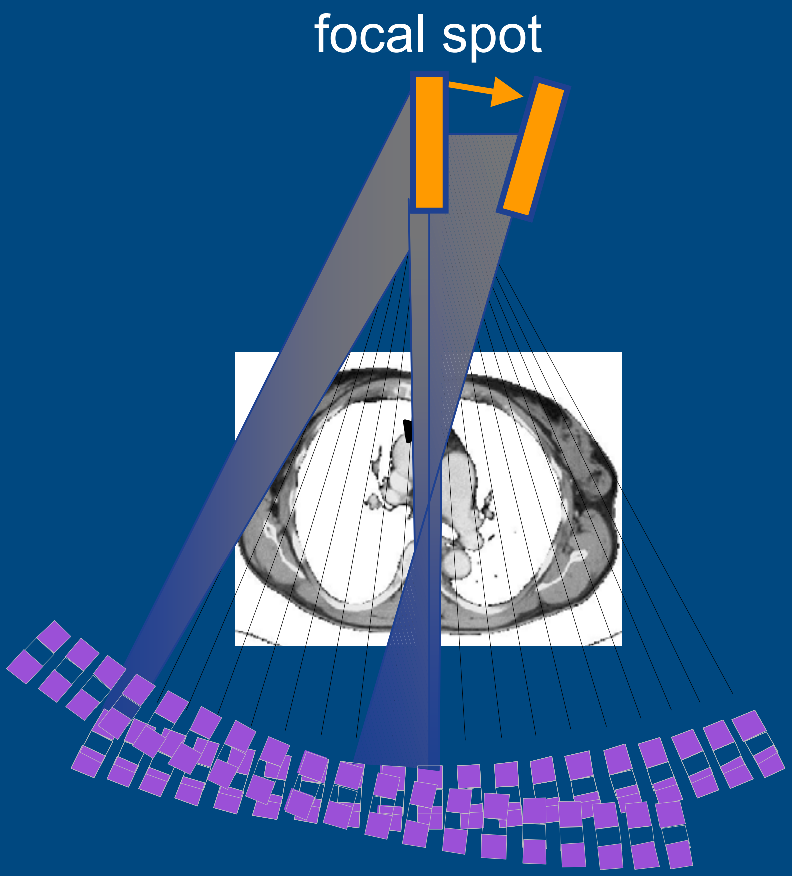
Finite beam width



Focal spot :
- Width (x)
- Thermal length
- Target angle
- Off-focal radiation

Detector cell :
- Cell size (in x and z)
- Cross-talk

Azimuthal blur :
- View duration
- Detector response time

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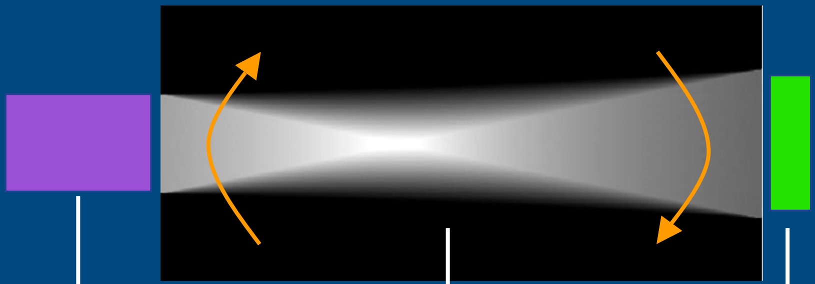
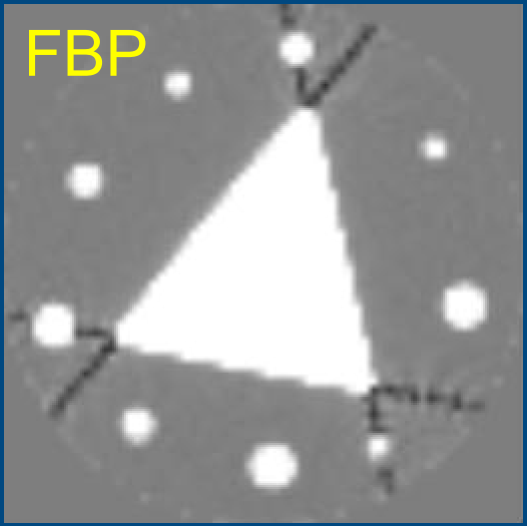
Incorporate physics : finite beam width



finite
focal spot

finite
rotation

finite
detector cell



$$\hat{y}_i = \sum_{s=1}^{S} \frac{b_i}{S} \cdot \exp(-\sum_{j=1}^{J} l_{ijs} \mu_j)$$

→ Re-derive update step

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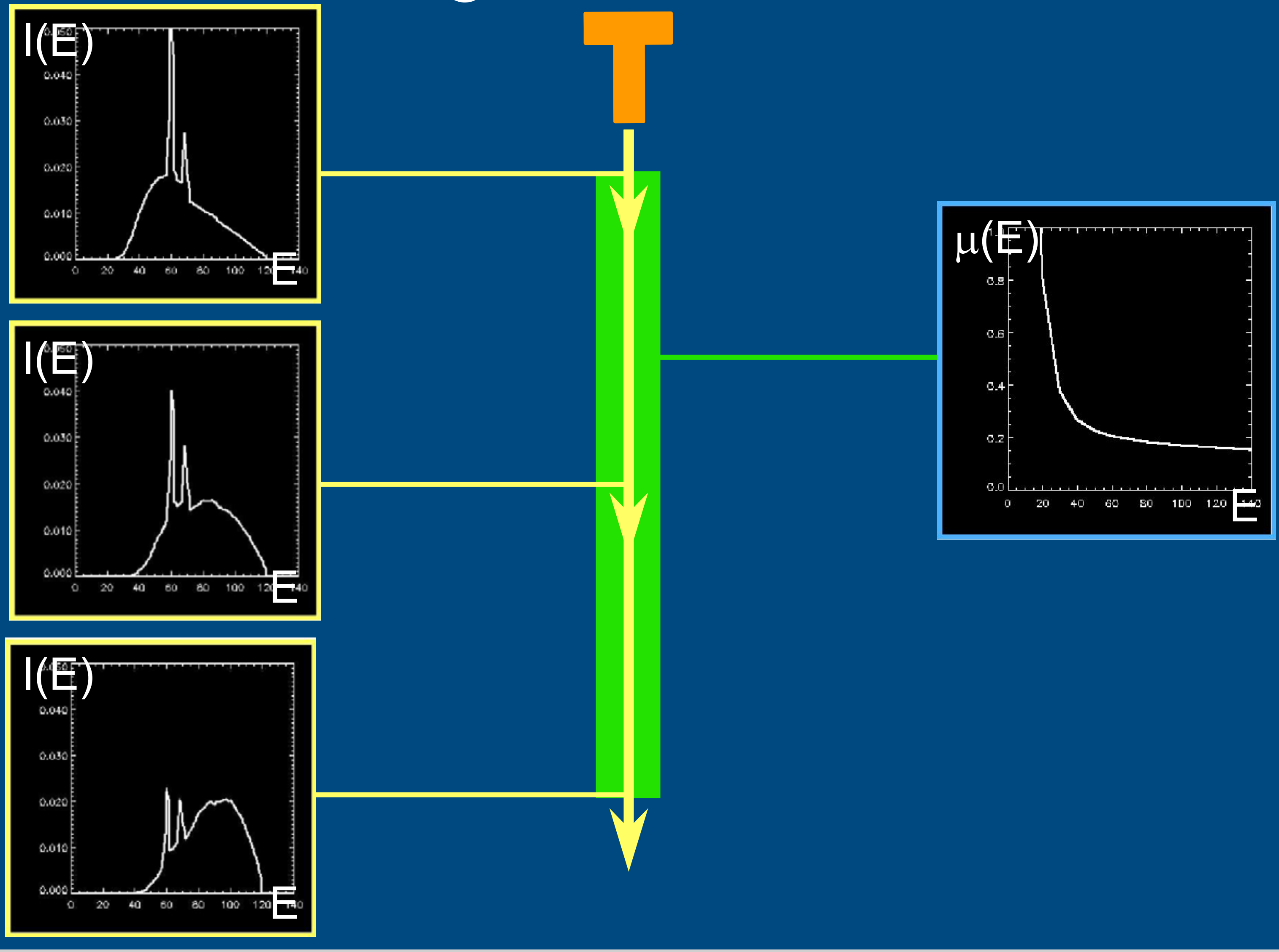
GE Global Research

B De Man, Ph.D. Thesis, 2001

Beam hardening



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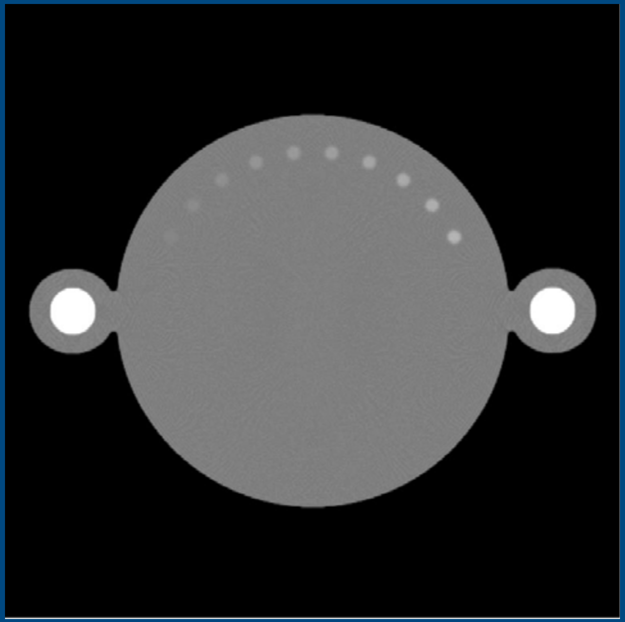
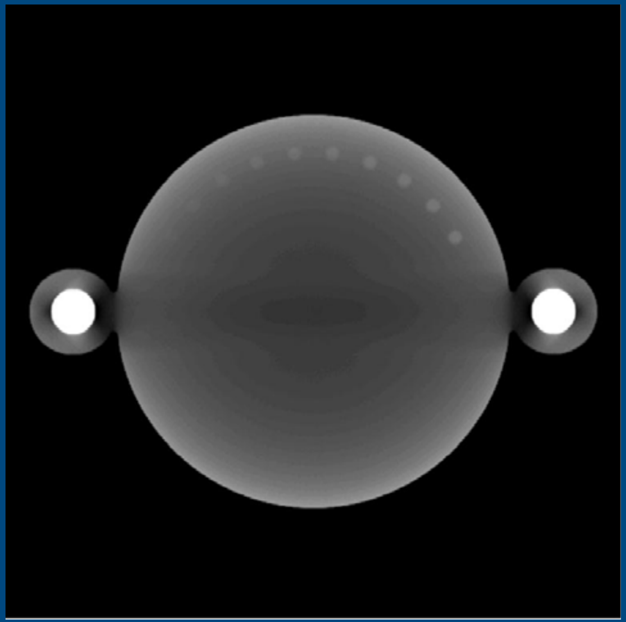
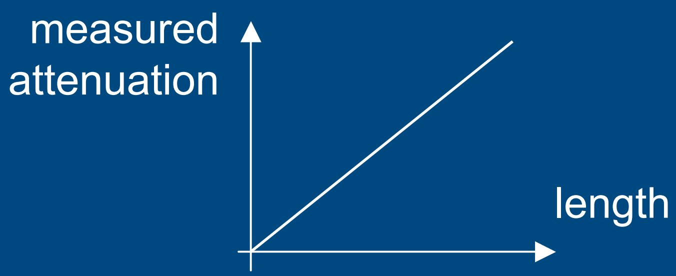
Beam hardening

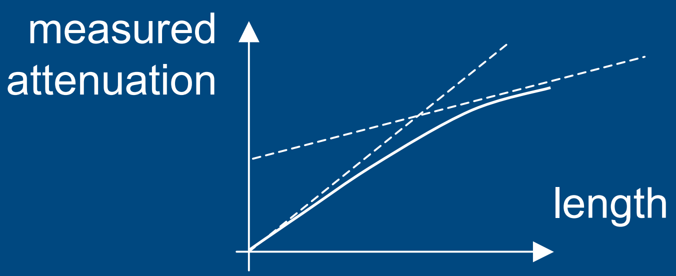
Monochromatic



Polychromatic







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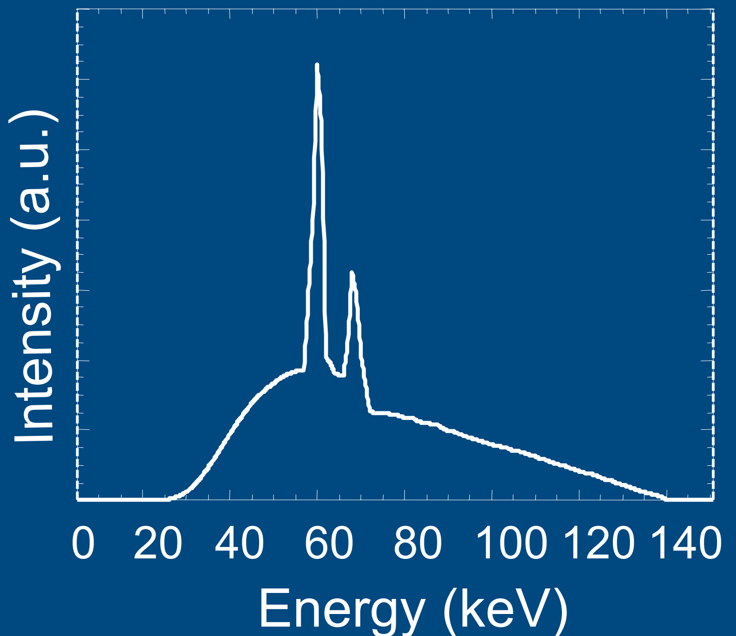
IEEE MIC, Honolulu, 2007

B De Man, IEEE Trans. Nucl. Sci., 1999

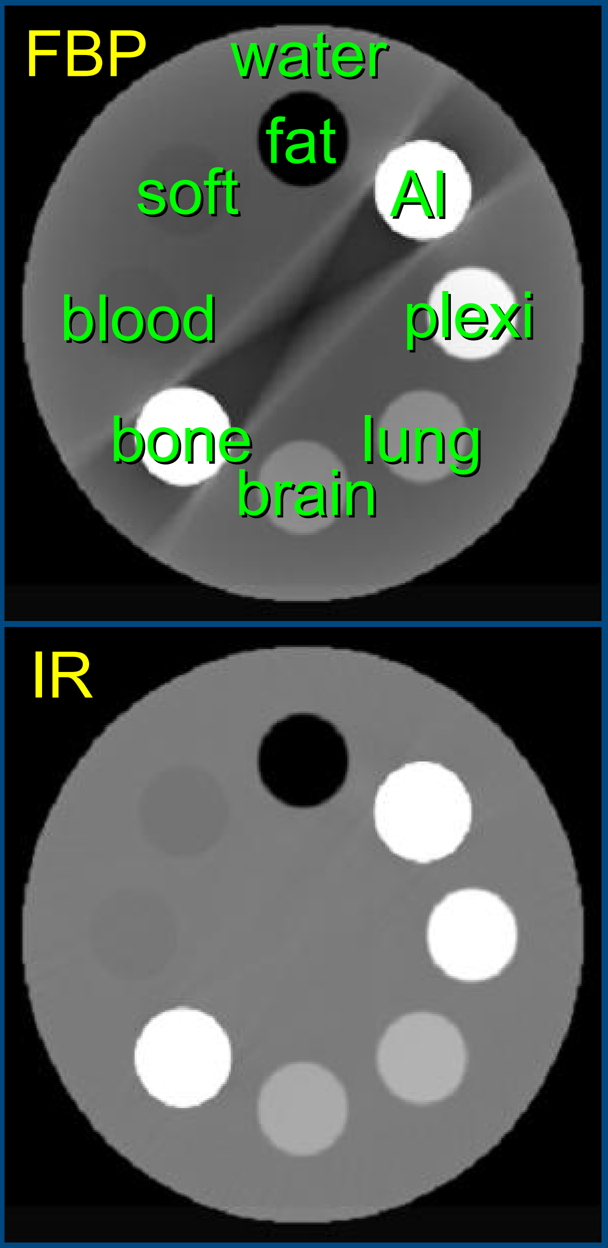
Incorporate physics : poly-chromaticity



$$\mu_j(E) = \phi_j \cdot \Phi(E) + \theta_j \cdot \Theta(E)$$

→ Need multi-energy measurements
OR constraint on ϕ_j, θ_j, μ_j

→ Re-derive update step



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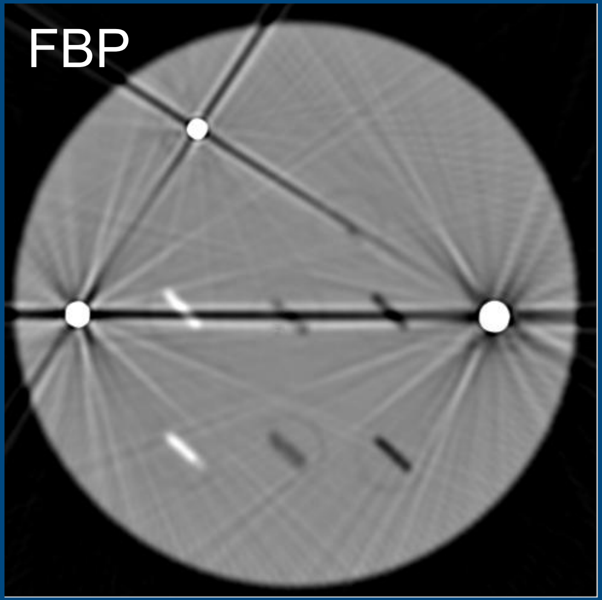
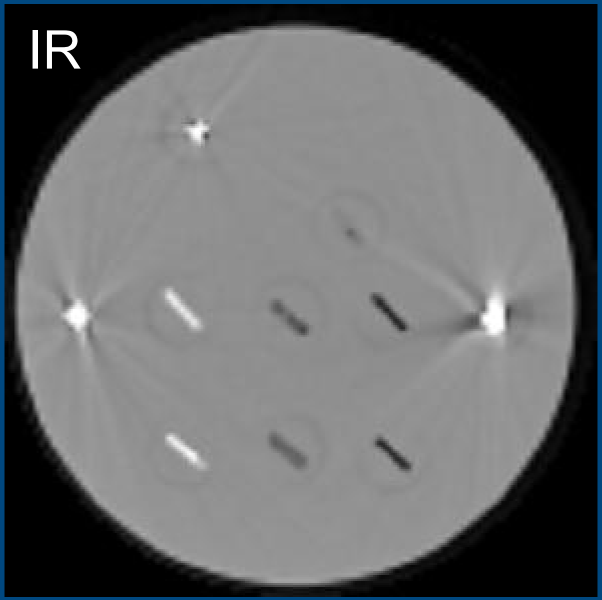
B De Man, IEEE Trans. Med. Im., 2001
I Elbakri, IEEE Trans. Med. Im., 2002

Missing data : MAR



1. treat all rays equally
2. pre-correct for physics



1. lower weight to corrupted rays
2. incorporate physics

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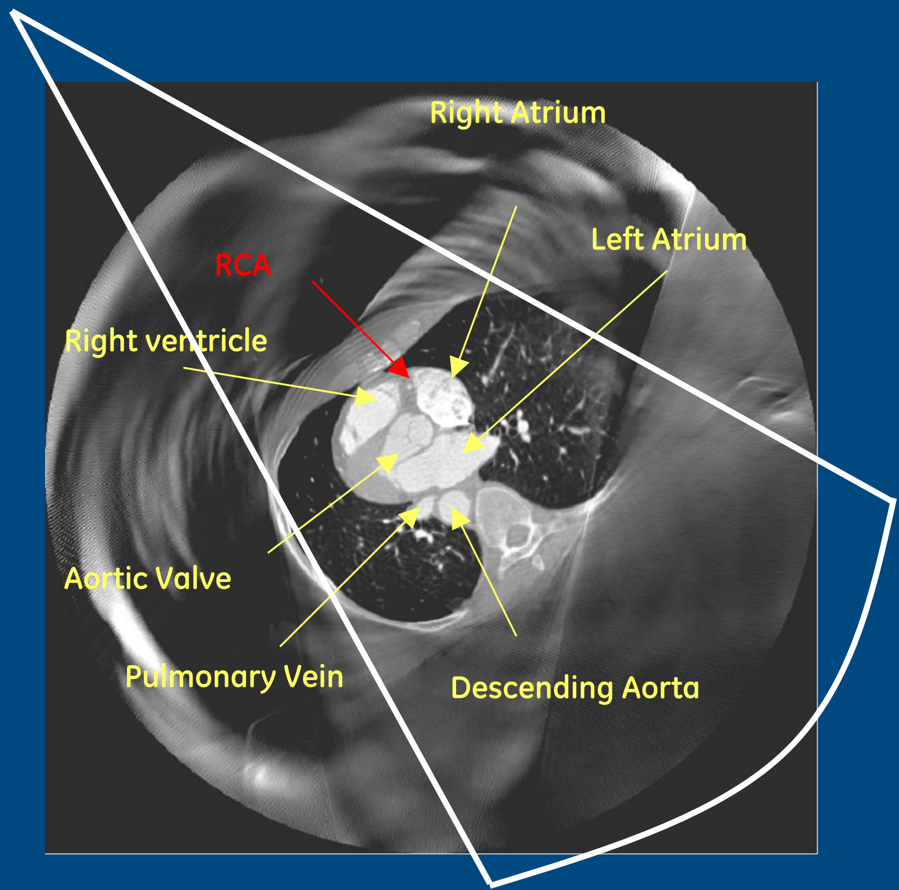
B De Man, IEEE Trans. Nucl. Sci., 2000

Missing data : truncation or local ROI



Exact reconstruction not possible for the pure local ROI case,
Exact reconstruction IS possible with some extra information

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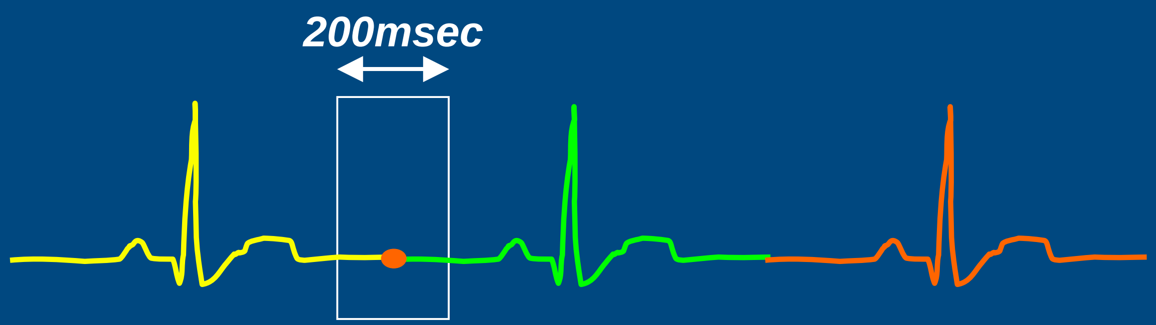
D Langan, SPIE, 2006

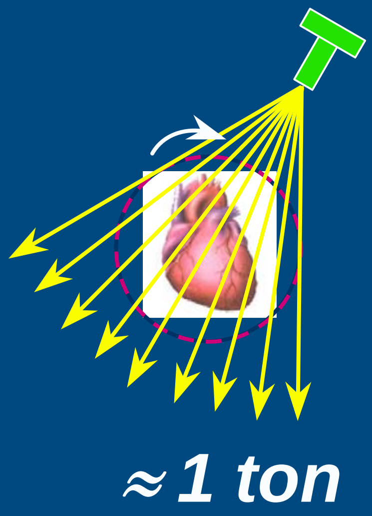
Dynamic imaging (cardiac CT)

rotation period : 0.35s

full-scan = 360° → half-scan ≈ 230°

temporal resolution : 200ms





Solutions : (1) spin faster
(3) multiple sources
(2) combine multiple heart beats
(4) motion compensation

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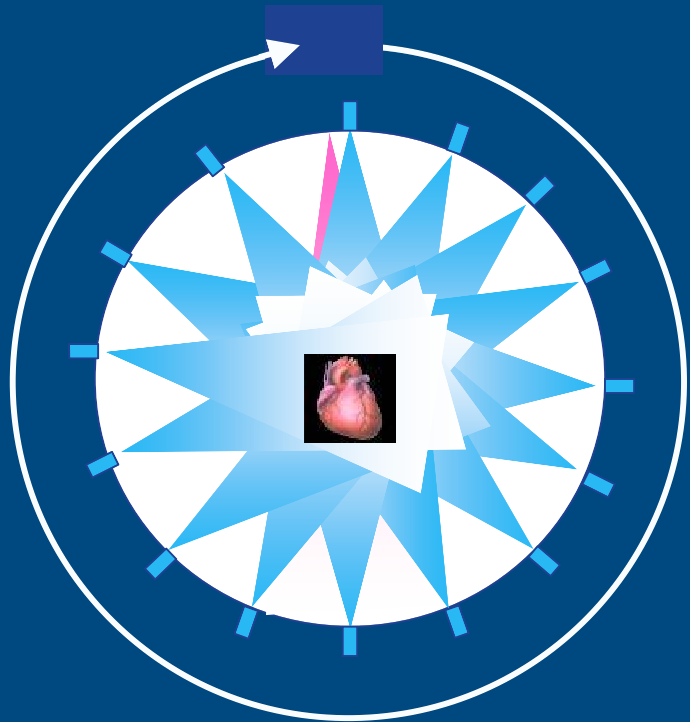
Experiment : slow-gantry cardiac CT

- 180bpm (rabbit heart)
- 1 full rotation
- 18s acquisition
- 1500 views
- 1 heart cycle = 1/3sec
- 54 heart cycles
- 28 phases / cycle
- temp.res. ~ 12ms

Slow rotation ~ 15s



Reconstruct each phase separately

↓

Estimate motion vectors

↓

Reconstruct 4D dataset by incorporating motion vectors

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IEEE MIC, Honolulu, 2007

B De Man, Fully 3D Meeting, 2005

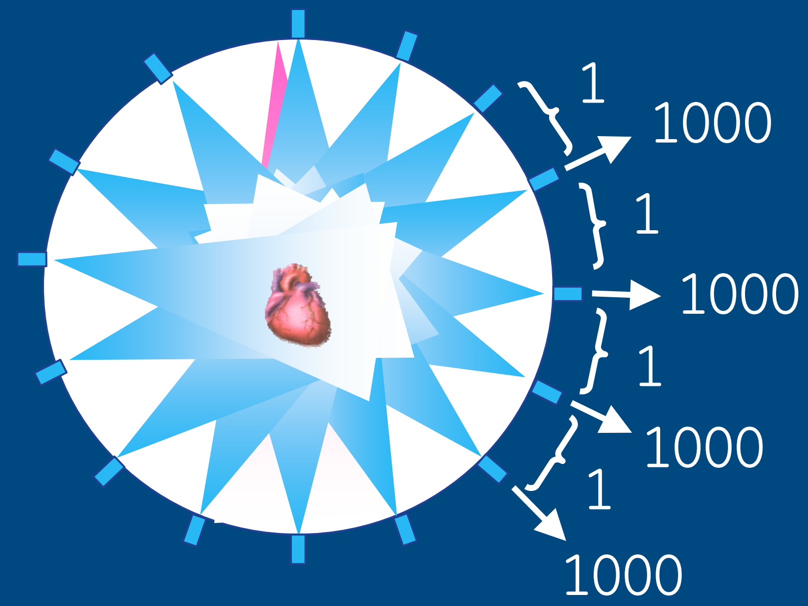
Phase-weighted IR

Rationale :

- Using data from 1 phase results in missing data
- Using all phases eliminates temporal info
- Therefore we should use phasic data if available and use other phases where phasic data is missing
- PWMLTR :

data $\in$ phase $\Rightarrow$ weight = 1000

data $\notin$ phase $\Rightarrow$ weight = 1



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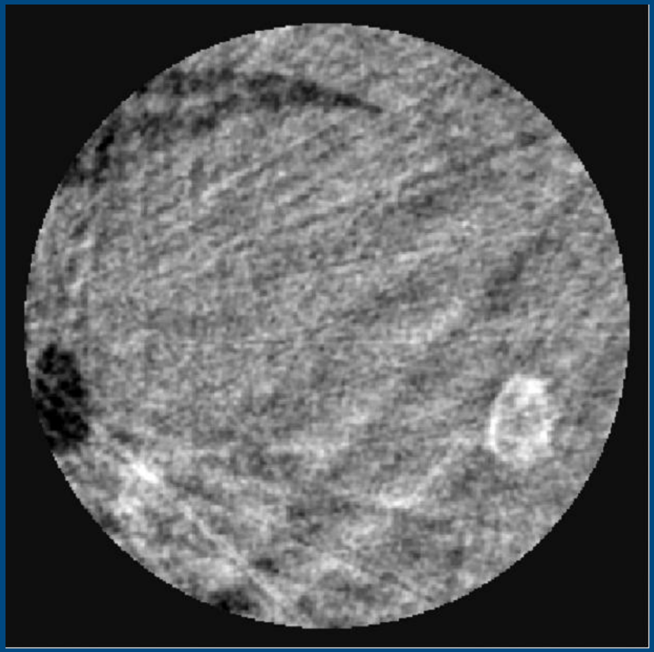
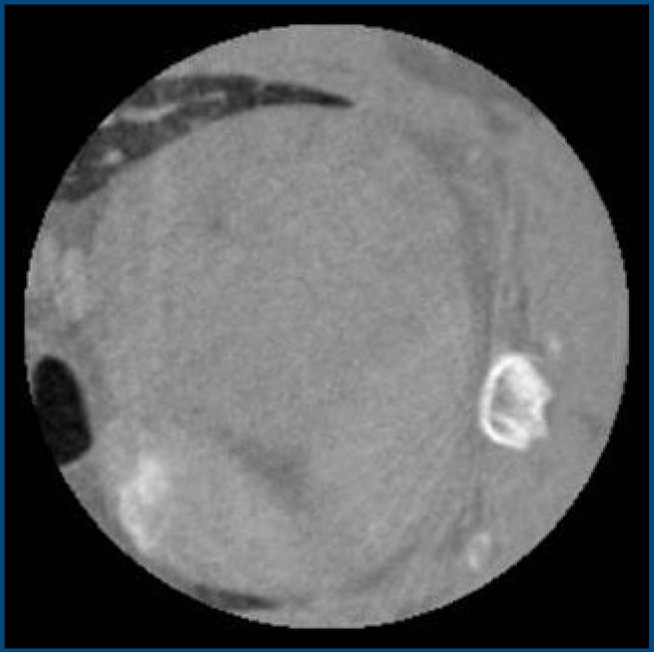
Experiment : slow-gantry cardiac CT

FBP



PW-MLTR



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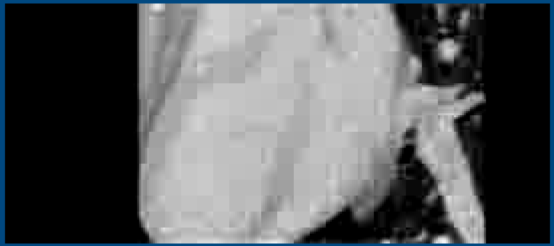
Experiment : slow-gantry cardiac CT



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