Iterative FDK reconstruction for helical CT of the head with rigid motion compensation

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***Abstract*—In previous work, we have developed a reconstruction algorithm for helical CT which compensates for a known rigid motion of the object being scanned [1]. This algorithm was used for CT imaging of the head, as part of a method which estimates possible rigid head motion on a view by view basis from the CT data themselves, and compensates for that motion during reconstruction of the clinical image. In this earlier work, we have used iterative maximum likelihood algorithms for the reconstruction. Compensating for motion in such algorithms is easy and effective, but the computation time is high. Here we propose an iterative FDK algorithm as an alternative. To compensate for rigid motion, we account for the motion during the backprojection step, while the filtering step is not modified. The resulting image suffers from low frequency artefacts. However, it is found that these artefacts are eliminated by applying a few iterations of the FDK reconstruction. The performance of the algorithm is demonstrated on a clinical CT scan which was affected by significant patient motion.**

I. Introduction

Compensating for rigid object motion in iterative reconstruction is fairly straightforward: the inverse motion can be assigned to the CT system by superimposing it on the system's helical trajectory. This results in an irregular trajectory of the source and detector in a coordinate system where the object is stationary. Iterative maximum likelihood (ML) algorithms use the original views, which is convenient because it is assumed that the pose of the object may be different for each view. In addition, they make use of all the measured data, which is beneficial for the signal to noise ratio of the reconstructed image. Very promising results have been obtained with this approach, and we are currently evaluating it for clinical application [1]. However, the long computation times of ML reconstruction hamper its introduction in the clinic. In this paper, we report on our attempt to use the FDK algorithm [2] as an alternative for ML reconstruction in this context. Advantages of the FDK algorithm are that it is fast, it uses the acquired views without the need for extensive interpolations (like e.g. fan- to parallel rebinning) and it can use (almost) all of the acquired data. Other choices are possible. Ye et al. proposed an analytical algorithm for cone beam reconstruction with an arbitrary trajectory, where the only requirement is that every point is located on a PI-line [3]. Jang et al. developed a motion correction approach with similarities to that of [1], where they replaced ML reconstruction with a faster analytical reconstruction [4]. That algorithm involves fan- to parallel rebinning, and they managed to account for the motion in this rebinning step.

We are evaluating our methods on different Siemens CT systems, the most recent one being the Siemens Force. On this system, the head CT scans are typically acquired with a low pitch (0.55), one rotation per second, 4200 views per rotation and flying focal spot (i.e. increasing the transaxial and axial sampling by small motions of the focal spot). In addition, the Force detector has non-uniform angular sampling, because the detectors are combined in modules, and there are small gaps between these modules. In the next sections, the preprocessing of the data to account for these sampling issues is briefly explained. Then application of the FDK algorithm for reconstructing scans without motion correction and its extension for motion correction are discussed. Finally, results obtained on a motion corrupted clinical CT scan are shown

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II. Methods

A. FDK for helical CT reconstruction unaffected by motion

1) Interpolation of adjacent views: To exploit the sampling improvement provided by the transaxial flying focal spot, pairs of adjacent views are combined into a higher resolution view, creating a new sinogram with twice the number of detector pixels, half the number of angles and an unchanged number of detector rows. The focal spot motion is designed to create a uniform sampling pattern of double density near the center of the field of view in the transaxial plane. Because of the fan angle geometry, this ideal interference is only obtained in the center, the interference deviates more from the ideal double sample at locations which are at larger distance from the center. This is illustrated by the red and blue lines in figure 1, which represent the two projection lines associated with two adjacent views. In that same figure, the gray lines represent the geometry of a virtual view with double resolution, which is to be computed (approximately) from the two available views. This virtual view is computed by interpolating the detector values of the two available views, according to the 1D coordinates on a line through the center and perpendicular to the central virtual projection (the dashed black line in fig. 1). This 1D coordinate assigned to each detector value is the position of the intersection point of the projection line associated with that detector and the central line. In other words, we assume that the available detector values correspond to the intersection of the red and blue lines with the black line in figure 1, and the target virtual detector values correspond to the intersection of the gray lines with the black line. For head

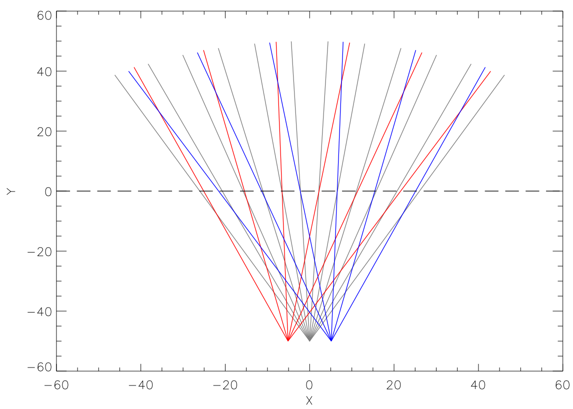

Fig. 1. The red and blue lines represent the geometry of a pair of adjacent views, which not only differ by a small rotation angle, but also by an additional opposite motion of the focal spot. The gray lines represent the geometry of a virtual detector with double detector sampling. The interpolation is done as if the detector values correspond to the intersection of the projection lines with the central dashed black line.

imaging, this approximation should be acceptable because the head should be positioned near the center of the field of view. If the CT system has in addition a non-uniform detector sampling, then that sampling is accounted for; for the virtual detector, always a uniform sampling is used.

2) Tam-Danielsson window: By masking the detector values with the Tam-Danielsson window, one can ensure that the remaining data provide projection values over exactly 180 degrees for each voxel (see e.g. [5]). The boundaries of the window are obtained by computing the projection of the locus of the source trajectory on a particular view. Figure 2 shows a view from a patient scan, together with a virtual view computed as outlined above and the Tam-Danielsson window for that view.

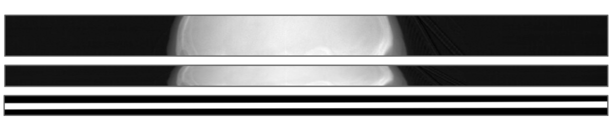

Fig. 2. Top row: A single view from the Siemens Force CT image. Middle row: a high resolution virtual view computed from two adjacent views. Bottom row: the Tam-Danielsson window for the virtual view (black values are pixels that are masked to zero).

Because of the low pitch, the view contains enough data for the application of a larger Tam-Danielsson window covering 3π of data for every voxel. Because with this window more of the data are used, the reconstructed image will have a better signal to noise ratio than when the π window were used. This is illustrated in fig. 3, which also shows that for a pitch of 0.55, the 3π window covers nearly the entire detector and therefore causes only a small loss of data.

3) FDK reconstruction: The FDK algorithm has been developed for circular cone beam CT, in which case it is exact for the central axial slice, and produces excellent images for the other slices, where exact reconstruction is not possible due to data insufficiency. Introducing some approximations, the FDK algorithm can also be applied to helical CT data.

Fig. 3. The Tam-Danielsson window of fig. 2 is shown in panel A, after changing the dimensions to make it more visible. Panel B shows the corresponding boundaries, obtained by projecting the source trajectory. Panel D shows that also the previous and next turns of the helical trajectory project on the detector, producing a Tam-Danielsson window that provides 3π of data for every voxel (panel C).

For CT reconstruction from scans acquired with a cylindrical detector, the FDK algorithm involves

1) multiplying the views with a position dependent weight,
2) filtering the projections with a modified ramp filter along a line tangent to the source trajectory,
3) computing a weighted backprojection of the filtered projections, where the weight decreases quadratically with the distance to the source,
4) zeroing the negative values.

The first step is straightforward. Considering that the view is very large and the pitch very small, as illustrated in fig. 2, the line tangent to the source trajectory is almost horizontal. Therefore, the filter is simply applied along the rows. The third step would require a modification of the backprojection procedure, if a volume based backprojector was used. However, we use a ray tracing backprojector (the adjoint operator of a ray tracing forward projector), which inherently has the desired backprojection weighting. We included the fourth step to impose the known non-negativity of the reconstructed attenuation coefficients.

We find that applying our current implementation of this procedure to the views masked with the 3π Tam-Danielsson window produces images of high resolution and good visual quality, although they suffer from subtle position dependent scaling artefacts of low spatial frequency, causing smooth variations of up to 10 HU. The artefacts rotate as one scrolls through the transaxial slices. These artefacts may be due to inaccuracies in our discretisation of the Tam window, because as shown below, even small distortions of the window can lead to significant scaling artefacts.

4) Iterative FDK reconstruction: The FDK algorithm can be extended to an iterative algorithm with the following heuristic approach:

$$
\begin{aligned}
X^{(0)} &= \mathrm{FDK}(Y) \\
X^{(i)} &= X^{(i-1)} + \mathrm{FDK}(Y - \hat{Y}(X^{(i-1)})), \qquad (1)
\end{aligned}
$$

where $X^{(i)}$ is the reconstruction at iteration i, FDK represents the non-iterative reconstruction described above, Y is the CT sinogram and $\hat{Y}(X)$ is the sinogram computed by forward projection of the image X. Applying one iteration eliminates the low frequency artefacts. Iterative FDK does not have proven convergence, but in simulations using noise-free data and applying 5 iterations, the sum of squared differences between the consecutive reconstruction images and the ground truth image decreased rapidly and monotonically.

B. FDK reconstruction with motion correction

To incorporate the motion in the FDK reconstruction, we simply model it during the backprojection step, without modifying the filtering.

The FDK algorithm is obtained by discretization of a backprojection integral, i.e. an integral over the rotation angle. As a result, every view has to be multiplied with the angular increment associated with each view. For a regular trajectory, this value is a constant, but for an irregular trajectory, this angular increment is view dependent. FDK was derived for a circular orbit and therefore it is not obvious how to deal with rotations in 3D. We solve this by projecting the motions to the transaxial plane. A 3D vector pointing from the source to the center of the detector is considered. This vector is projected to the transaxial plane and we compute how much incremental 2D rotation it undergoes from one view to the next. The angular increment assigned to each view is computed as the mean of the angular increment between the previous view to the current one and the increment between the current view and the next.

The Tam-Danielsson window is valid for a helical trajectory, but not for a distorted trajectory. Due to the motion, the window no longer ensures that exactly 3π of tomographic data are used for every voxel. The resulting use of insufficient or excessive redundant data produces a very significant position dependent scaling, even for small motions. In addition, the Tam-Danielsson window has sharp boundaries, which can produce sudden changes in the local scale factors, in particular in the direction of the rotation axis. This is illustrated in figure 4 for a clinical CT study affected by patient motion. The patient moved significantly when the lower part of the head was scanned, but motion correction was done for the entire scan. Fig. 4 shows that also the very small motion corrections for the other parts of the scan resulted in very disturbing artefacts.

As a partial remedy, the 3π window was replaced with a smoothed version of the π window, illustrated in figure 5. The smoothing was done in the vertical direction with a Gaussian convolution mask, designed to ensure that the resulting window covers all detector rows. The integral of the convolution mask equals 1, which ensures at least that the decreased weight of the central π data is compensated by the use of redundant data outside the π window. This window does not provide exact weights and results in a degradation of the reconstruction in the motion-free case. On the other hand, in the motion case it may be beneficial, because it eliminates the high frequency streaks that were due to the sharp transitions in the unsmoothed Tam-Danielsson window.

To eliminate the scaling artefacts, a few iterations of the FDK algorithm are applied (eq (1)). The original FDK images and the first two FDK iterations are shown in figure 6 for the clinical motion corrupted study, using a narrow display window of about 70 HU to clearly show the scaling artefacts. The first iteration strongly suppresses and the second iteration eliminates the scaling artefacts.

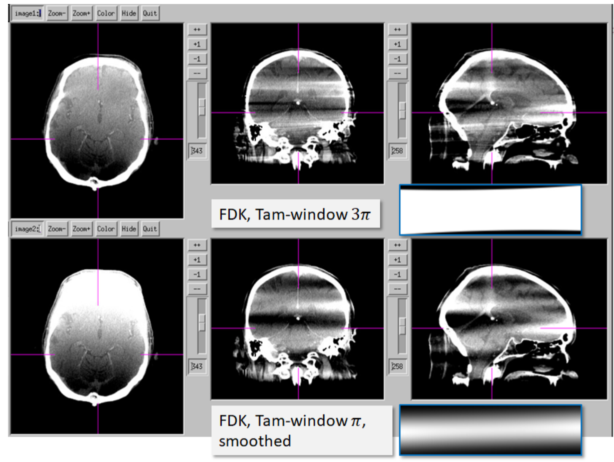

Fig. 4. When the 3π Tam-Danielsson window is used for reconstruction with motion correction, the reconstructed image suffers from artefacts, which are low frequency in the transaxial plane but can have high frequencies in coronal or sagittal slices (first row). Using instead a smoothed version of the π window makes the artefacts low frequency in all dimensions.

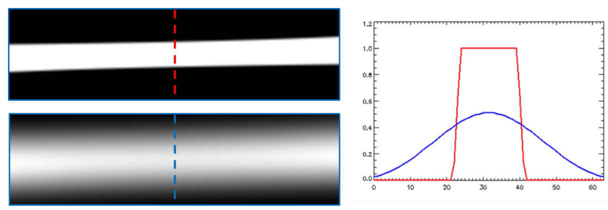

Fig. 5. The images show (resized versions of) the π Tam-Danielsson window and a Gaussian smoothed version of that. Profiles along the dashed lines are shown in the plot.

III. Results

Figures 7 and 8 show selected slices from two different scans, comparing the vendor reconstruction to the reconstruction with the proposed iterative FDK algorithm (2 iterations).

The slice of the first scan, shown in fig. 7, was affected by motion. The reconstructed images are shown with a bone window of [-250, 1250] HU. The vendor reconstruction suffers from significant patient motion artefacts, which are eliminated by FDK with motion compensation.

The slice selected from the second scan, shown in fig. 8, was not affected by motion. The reconstructed images are shown with a brain window of [-40, 110] HU. For brain imaging, the vendor software applies a dedicated bone beam hardening reconstruction which involves the forward projection of a bone only image. This correction is applied on top of the standard water-based correction. Therefore, we have implemented a similar method, using the first FDK reconstruction (iteration 0) to extract the bone image. Then the bone beam hardening correction is applied to the sinogram, which is used in the calculation of the first and second iteration of the FDK reconstruction. This means that the first FDK iteration is initialized with the FDK image obtained without bone beam hardening correction, but it is found that the images are similar enough for this to be an effective initialization. Figure 8 compares the vendor and the FDK reconstruction images. The pixel size of the slices is 0.47 mm x 0.47 mm,

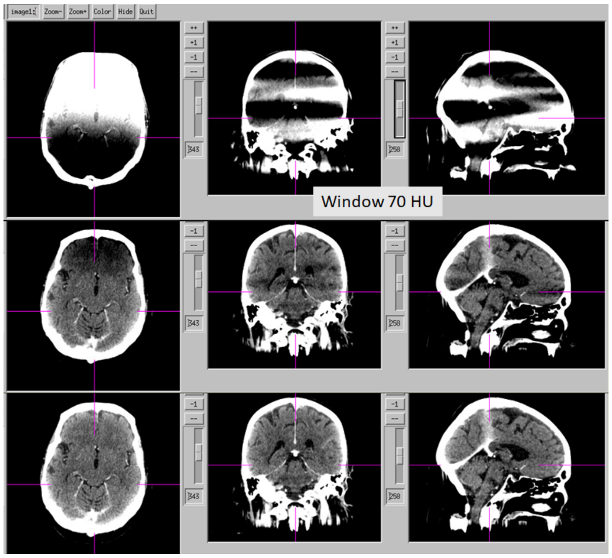

Fig. 6. The original FDK reconstruction with motion correction is shown in the first row, using a narrow display window of about 70 HU. The next rows show the first and second FDK reconstructions using the same display window.

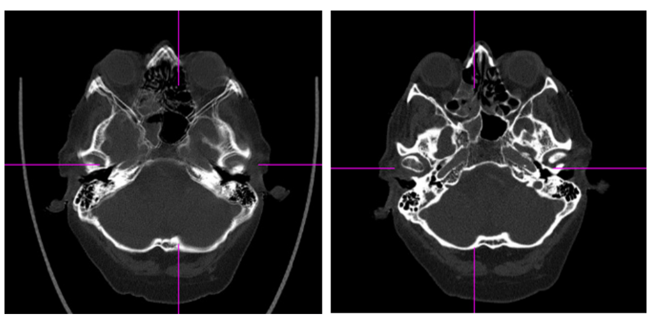

Fig. 7. The vendor reconstruction (left) and the iterative FDK reconstruction (right) from a motion corrupted CT scan. This is the same scan as the one used in figures 2, 4 and 6. A bone display of [-250, 1250] HU was used.

with a slice separation of 0.75 mm. There is a small axial mismatch between the images of less than half the slice separation. The images are not identical, which may due to small differences in the beam hardening correction and the use of a different post-reconstruction filter. We have post-filtered the FDK reconstruction with a Butterworth filter, trying to match the features of the vendor image. Nevertheless, we think that the images are rather similar and probably have very similar diagnostic value.

IV. Discussion

FDK is an effective reconstruction algorithm developed for circular cone beam CT. We have implemented an iterative version of the algorithm for helical CT with motion correction. Our results show that good images are obtained, both for bone and brain imaging. Even with 2 iterations, FDK is still much faster than ML-reconstruction for CT. When bone beam hardening correction is applied, the vendor reconstruction uses two backprojections and one forward projection, whereas the proposed approach needs three backprojections and three forward projections. If code with similar efficiency would be

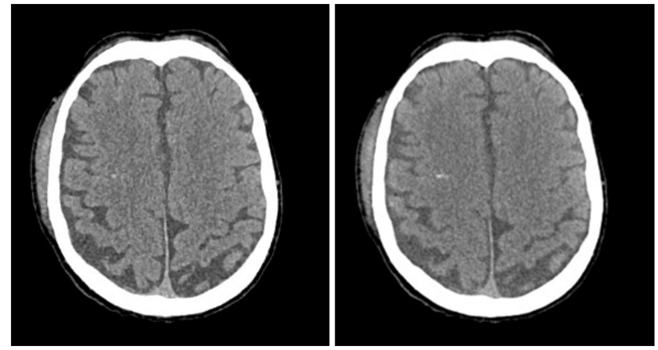

Fig. 8. The vendor reconstruction (left) and the iterative FDK reconstruction (right) obtained from a scan portion unaffected by motion. Both reconstruction are done with an additional bone beam hardening correction, which is required to obtain a uniform HU distribution inside the skull. The slices are approximately matched (there can be a mismatch of about 0.35 mm in axial position).

used, the computation time should only be about twice as long. Therefore, we believe this will help in introducing data driven motion correction into the clinical routine.

Acknowledgment

The authors would like to thank Frédéric Noo, Michel Defrise and Karl Stierstorfer for helpful discussions, and the latter also for his help with reading and understanding the Siemens CT data.

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