GEOMETRICAL CALIBRATION AND APERTURE CONFIGURATION DESIGN IN MULTI-PINHOLE SPECT

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ABSTRACT

A clinical gamma camera can be converted into a high resolution SPECT system for small animal imaging, by replacing the clinical collimator(s) with pinhole collimators. However, for optimal performance, an accurate geometrical calibration is required. If it is assumed that the detector orbit is a true circle, the calibration requires the determination of seven parameters. It has been shown that these can be uniquely determined from a SPECT scan of a phantom consisting of three point sources, if two of the distances between these point sources are known. For multi-pinhole SPECT, two point sources should be sufficient, knowledge of the distance between the point sources is not required. The calibration method has been extended for cases where the orbit deviates from the ideal circle. A second interesting problem in multi-pinhole SPECT is the optimisation of the collimator design. This requires a measure of image quality, enabling objective comparison between different designs. An efficient analytical method has been developed for that purpose. With this method, it has been shown that the aperture diameter should be slightly smaller than the desired system resolution. We have also found that increased multiplexing (which comes with an apparent increase in system sensitivity) does not lead to reduced variance for a particular target resolution. In practice, avoiding all overlap seems to yield better performance.

***Index Terms*—** pinhole SPECT, calibration, multiplexing, microSPECT, small animal imaging

1. INTRODUCTION

This paper discusses two problems encountered in micro-SPECT imaging: the geometrical calibration of a (multi-) pinhole SPECT system, and the optimisation of the collimator design. The calibration problem of pinhole SPECT is mathematically equivalent to the calibration of cone beam CT, so the results obtained for SPECT with a single aperture apply to cone beam CT calibration as well. The methodology that was used to study the pinhole collimator design is fairly general and can be applied to other system design problems in emission or transmission tomography.

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2. GEOMETRICAL CALIBRATION

The calibration problem is most relevant when the micro-SPECT system is based on a clinical rotating gamma camera. However, the results may also be of interest for dedicated rotating or stationary micro-SPECT systems. The calibration provided by commercial software on a clinical gamma camera is obviously sufficient for clinical applications at relatively low spatial resolution (5..10 mm and higher). However, the spatial resolution can be improved dramatically by using a pinhole collimator. To fully exploit the potential of such a collimator, a far more accurate calibration is required.

If it can be assumed that the orbit of the detector head is perfectly circular, the geometry of a single aperture system can be characterized with 7 parameters. A gamma camera detector has 6 degrees of freedom (3 rotations and 3 translations), a pinhole aperture has three degrees of freedom (3 translations). However, an initial rotation around the rotation axis of the circular orbit can be incorporated in the circular orbit by changing the starting angle. Secondly, the position along the rotation axis is arbitrary. Consequently, 7 parameters remain. We have used the following set of parameters (illustrated in fig. 1), some of which are defined with the help of the central ray, i.e. the line through the aperture and perpendicular to the detector. The focal distance f is the distance between aperture and detector, d is the distance between the aperture and the rotation axis, measured along the central ray, the mechanical offset m is the distance between the rotation axis and the central ray, e_u and e_v are the electronic offsets of the detector, the tilt ϕ is a rotation in a plane parallel to the rotation axis and the central ray, and the twist ψ is a rotation of the detector around the central ray. The mechanical aperture offset n parallel to the rotation axis is set to zero by an appropriate choice of the origin. It has been shown that these parameters can be uniquely determined from a SPECT scan of a phantom consisting of three point sources, if two of the

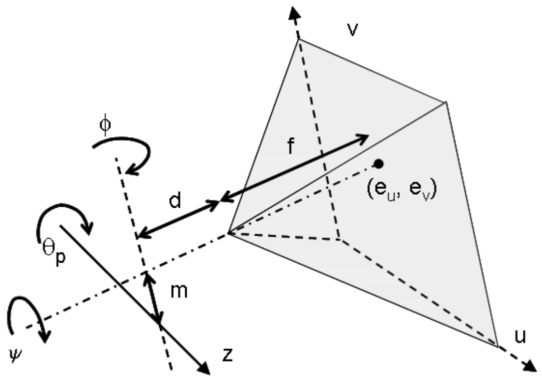

Fig. 1. The geometrical parameters of a pinhole system with ideal circular orbit.

distances between these point sources are known [1]. Wang and Tsui have shown that for a multi-pinhole system, it is sufficient to use two point sources, while the distance between the sources is not required [2]. An equivalent derivation using the parameterisation of [1] is given in the appendix. In these procedures, the projections of the point sources are described as a function of the calibration parameters, the point source positions and the projection angle:

$$[u_p, v_p]' = F(f, e_u, e_v, m, d, \phi, \psi, \vec{x_1} \ldots \vec{x_I}, \theta_p), \quad (1)$$

where θ_p is the angle of projection p and I is the number of point sources. The prime denotes the matrix transpose. The calculated projections $[u_p, v_p]'$ are fitted to the centers of the measured projections of the point sources to determine the values of all parameters (including the "nuisance" parameters $\vec{x_i}$). Wang and Tsui reported good results. In our limited experience, it seems though that the stability of the calibration may be improved by adding the distance between the two sources if it is available. A more rigorous study of the stability is required.

In practice, it is found that the orbit of clinical gamma cameras often deviates from the ideal circle. The deviations are small and irrelevant for clinical applications, but in pinhole SPECT they are found to degrade the resolution. Defrise et al have shown that the calibration scan of the three point sources provides enough information to compute a useful estimate of these deviations [3]. The deviations are assumed to be a rigid motion of the detector-collimator assembly. Assuming that the deviations are small, their effect can be described using first order approximations:

$$[\Delta u_p, \Delta v_p]' \simeq M[\Delta x_p, \Delta y_p, \Delta z_p, \Delta\psi_p, \Delta\phi_p, \Delta\theta_p]', \quad (2)$$

where p is the index of the projection, and (x_p, y_p, z_p) is the position of the detector-collimator assembly. The differences $[\Delta u_p, \Delta v_p]'$ are the residuals obtained when a perfect circle is assumed. The deviations of the geometric parameters can be estimated by solving (2). However, the matrix M is rectangular, and may not be of maximum rank. For that reason, an approximate inverse is calculated using singular value decomposition, discarding small singular values [3]. Correction for these deviations yielded a noticeable improvement of the spatial resolution for the system under study.

Fortunately, it appears that the orbit of the gamma camera is very reproducible. Consequently, the calibration parameters can be determined from a separate SPECT scan. In our scanning protocol, first the small animal SPECT scan is performed. Then, the animal is replaced with the point sources phantom and the SPECT scan is repeated. Others have argued that several of the parameters are stable over time, so there is no need to calibrate them repeatedly. Fixing some of the parameters reduces the minimum number of point sources required for calibration [4]. E.g., when the focal distances f_j for the apertures $j = 1 \ldots J$ in a multi-pinhole system are known, calibration only requires a single point source. In some applications, it may be possible to put the point source(s) in the field of view when scanning the small animal, eliminating the need for a separate calibration scan.

3. PINHOLE COLLIMATOR DESIGN

Assume that one wishes to compare two different pinhole systems, one with system matrix A and the other one with $\tilde{A}$. In a thought experiment, both systems can be used to acquire a sinogram of exactly the same activity distribution λ. Both sinograms will be corrupted by Poisson noise, the expected value of the sinograms will be

$$E(y) = A\lambda \quad \text{and} \quad E(\tilde{y}) = \tilde{A}\lambda. \quad (3)$$

The log-likelihood function for this problem equals

$$L(y|x) = \sum_i y_i \ln(Ax) - Ax. \quad (4)$$

Both sinograms can be reconstructed with a maximum-a-posteriori algorithm (MAP) to yield the images x and $\tilde{x}$. Applying a second order series expansion, Fessler et al [5, 6] and Qi et al [7] have analyzed the linearized local impulse response and covariance of images reconstructed with MAP using a quadratic smoothing prior. Assume that the tracer uptake at the position corresponding to pixel j in the image is changed with a small amount, from λ into $\lambda + e^j$. The expectation of the sinogram then changes into $E(y) + Ae^j$ (and similar for $\tilde{y}$). The local impulse response in pixel j and its covariance matrix then become

$$\begin{aligned} l^j(x) &= E(\text{MAP}(y + Ae^j)) - E(x) \\ &\simeq [F + \beta R]^{-1} F e^j \quad (5) \\ \text{Cov}^j(x) &\simeq [F + \beta R]^{-1} F [F + \beta R]^{-1} e^j, \quad (6) \end{aligned}$$

where R is the second derivate of the quadratic smoothing prior, β is the weight assigned to the prior. F is the Fisher information matrix which can be written as

$$F = A' \,\text{diag}[E(y)]^{-1} A, \quad (7)$$

where diag$[c]$ is a diagonal matrix with the elements of column matrix c on its diagonal.

To compare the two systems, they are forced to produce reconstructed images with a predefined target resolution [8]. The system that produces these images with the lower variance is then considered to be superior. To impose the target resolution, we assume that a prior can be designed such that

$$l^j(x) \simeq [F + \beta R]^{-1} F e^j \simeq T e^j, \tag{8}$$

where T is the target point spread function. For the covariance, we then obtain

$$\mathrm{Cov}^j(x) \simeq T F^{-1} T'. \tag{9}$$

For pinhole SPECT with a circular orbit, the data are insufficient, implying that F^{-1} does not exist. For that reason, the expressions have to be modified using an approximate inverse [8]. Assuming local shift invariance, the local impulse response and covariance near pixel j can be well approximated using shift invariant filters applied in the Fourier domain [7]. If only a small set of voxels is studied, then this analytical approach is orders of magnitude faster than estimating the (co)variances from multiple noise realisations. This method is general, and has e.g. been used to analyze the variance in PET with time-of-flight [9].

3.1. Aperture diameter

The analytical method was applied to find the optimal aperture diameter for a particular target resolution [8]. It was found that the aperture needs to be slightly smaller than the intended resolution. There are two ways to reduce the variance: using a larger aperture diameter to improve the sensitivity, and smoothing the reconstructed image. It appears that the optimal result is obtained with a combination of both: the aperture has to be smaller than the intended resolution, such that there is room for some smoothing.

3.2. Multiplexing

The sensitivity of a pinhole system can be improved by adding more apertures. Many groups have experimented with so-called multiplexing multi-pinhole systems: each aperture forms a projection onto the detector, and in these systems, the projections of different apertures are allowed to overlap. Allowing overlap increases the number of detected photons. However, it is not obvious that it also increases the effective sensitivity. In systems without overlap, every detected photon provides information about a single line integral through the activity distribution. In systems with overlap, some detected photons only provide information about a group of line integrals. Consequently, the reconstruction task is more complex, which is expected to increase the variance.

It has been reported that the variance decreases with increasing number of apertures up to a certain point; adding more apertures still increases the number of detected photons, but without a further decrease in variance [10]. The addition of extra apertures then needlessly complicates the system and the reconstruction. Moreover, with more overlap, the reconstructions are more prone to artifacts, in particular when the apertures are organized in a regular pattern [10].

This problem has been studied with the analytical method [11], applying it to the design of a collimator for rat brain imaging. Again, it was found that the variance is minimal at a fairly low number of apertures per detector ($7 \ldots 9$); adding more apertures had virtually no effect on the variance.

In addition, for applications where all pinholes focus on the same field of view (FOV), we found that most overlap-free designs outperform designs with overlap. In particular, with overlap-free designs, the image quality in the center can be increased beyond what is achievable with overlapping designs. However, this increase comes at the cost of a decreased quality near the edge of the FOV.

4. SUMMARY

A clinical gamma camera can be converted into a micro-SPECT system by using dedicated pinhole collimators. This paper discussed the geometrical calibration of the system and the design of pinhole collimators. Even for irregular camera orbits, an accurate calibration can be obtained based on the scan of a simple point sources phantom. An algorithm has been developed to rapidly predict the resolution and (co-)variance of particular voxel values in the final reconstruction, as a function of the system design. This enables the evaluation of a large number of designs in a relatively short time.

5. APPENDIX

This appendix studies the calibration of multi-pinhole systems. The derivation is similar to that presented in [1] for a single pinhole system. The following equations hold for the projection of point source j with cylindrical coordinates (r_j, α_j, z_j) through pinhole aperture i at projection angle θ:

$$\begin{aligned}
u &= f_i(M_1 + Z_1 + R_1)/N + M_1 + e_u && (10)\\
v &= f_i(M_2 + Z_2 + R_2)/N + M_2 + e_v && (11)\\
M_1 &= m_i \cos\psi - n_i \sin\psi \\
M_2 &= m_i \sin\psi + n_i \cos\psi \\
Z_1 &= z_j \cos\phi \sin\psi \\
Z_2 &= -z_j \cos\phi \cos\psi \\
R_1 &= r_j(-\cos\gamma\cos\psi + \sin\gamma\sin\phi\sin\psi) \\
R_2 &= -r_j(\cos\gamma\sin\psi + \sin\gamma\sin\phi\cos\psi) \\
N &= d_i + R_j \cos\phi \sin\gamma - z_j \sin\phi \\
\gamma &= \alpha_j - \theta
\end{aligned}$$

In [1] it was shown that a single pinhole with a single point source provides sufficient data to determine ψ. This allows to precorrect for ψ, i.e. set $\psi = 0$, which simplifies the expressions. We assume that there are two solutions that can explain the data by assigning values to the calibration parameters $f_i, m_i, n_i, d_i, e_v, e_v, \psi$ and the nuisance parameters r_j, α_j and z_j. Proceeding as in [1], denoting the second solution with a tilde, and assuming $n_1 = 0 = \tilde{n}_1$, we obtain

$$\tilde{\psi} = \psi \tag{12}$$

$$\tilde{f}_i = f_i \frac{\cos\tilde{\phi}}{\cos\phi} \tag{13}$$

$$\tilde{\alpha}_j = \alpha_j \tag{14}$$

$$\frac{\tilde{n}_i}{\tilde{r}_j} + \frac{\tilde{d}_i \sin\tilde{\phi} - \tilde{z}_j}{\tilde{r}_j \cos\tilde{\phi}} = \frac{n_i}{r_j} + \frac{d_i \sin\phi - z_j}{r_j \cos\phi} \tag{15}$$

$$\frac{\tilde{d}_i - \tilde{z}_j \sin\tilde{\phi}}{\tilde{r}_j \cos\tilde{\phi}} = \frac{d_i - z_j \sin\phi}{r_j \cos\phi} \tag{16}$$

$$\frac{\tilde{m}_i}{\tilde{r}_j} = \frac{m_i}{r_j} \tag{17}$$

$$(\tilde{m}_i - m_i)\cos\psi - (\tilde{n}_i - n_i)\sin\psi + (\tilde{e}_u - e_u) = f_i \frac{\sin\psi}{\cos\phi}(\sin\phi - \sin\tilde{\phi}) \tag{18}$$

$$(\tilde{m}_i - m_i)\sin\psi + (\tilde{n}_i - n_i)\cos\psi + (\tilde{e}_v - e_v) = -f_i \frac{\cos\psi}{\cos\phi}(\sin\phi - \sin\tilde{\phi}) \tag{19}$$

$$f_i + d_i = R \quad \text{and} \quad \tilde{f}_i + \tilde{d}_i = \tilde{R} \tag{20}$$

For a system with two pinholes and a single point source and assuming $m_1 \neq m_2$, these equations can be solved for an arbitrary value of $\tilde{\phi}$. That solution is

$$\begin{aligned}
\tilde{f}_1 &= f_1 \frac{\cos\tilde{\phi}}{\cos\phi} \quad \text{and} \quad \tilde{f}_2 = f_2 \frac{\cos\tilde{\phi}}{\cos\phi} \\
\tilde{e}_v &= e_v + f_1 \frac{\sin\tilde{\phi} - \sin\phi}{\cos\phi} \\
\tilde{n}_2 &= n_2 + (f_2 - f_1) \frac{\sin\tilde{\phi} - \sin\phi}{\cos\phi} \\
\tilde{d}_1 &= \frac{d_1(1 - \sin\phi\sin\tilde{\phi}) + z(\sin\tilde{\phi} - \sin\phi)}{\cos\phi\cos\tilde{\phi}} \\
\tilde{z} &= \tilde{d}_1 \sin\tilde{\phi} - \cos\tilde{\phi} \frac{d_1 \sin\phi - z}{\cos\phi} \\
\tilde{d}_2 &= \tilde{d}_1 + \tilde{f}_1 - \tilde{f}_2.
\end{aligned}$$

Consequently, a single point source is insufficient to uniquely determine the geometrical calibration. Adding more apertures does not help, because each aperture adds three parameters and three equations. Fixing the distance between apertures only helps if $f_1 \neq f_2$, which will not be the case in many designs. However, when a second point source is added with $z_2 \neq z_1$, it is easy to show that $\phi = \tilde{\phi}$, leading to a unique solution. The same result has been obtained earlier by Wang et al [2] using a different parameterisation of the geometry.

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